Chapter 0: Algebraic Concepts

Exercises 0.1

1. $12 \in \{1, 2, 3, 4, \ldots\}$

2. $5 \notin \{x: x \text{ is a natural number greater than } 5\}$

3. $6 \notin \{1, 2, 3, 4, 5\}$

4. $3 \notin \varnothing$

5. $\{1, 2, 3, 4, 5, 6, 7\}$

6. $\{7, 8, 9\}$

7. $\{x: x \text{ is a natural number greater than 2 and less than 8}\}$

8. $\{x: x \text{ is a natural number greater than 6}\}$

9. $\varnothing \subseteq A$ since $\varnothing$ is a subset of every set. $A \subseteq B$ since every element of A is an element of B. $B \subseteq B$ since a set is always a subset of itself.

10. $\varnothing \subseteq A$ since $\varnothing$ is a subset of every set. $A \subseteq B$ since every element of A is an element of B. $B \subseteq B$ since a set is always a subset of itself.

11. No. $c \in A$ but $c \notin B$.

12. No. $12 \in A$ but $12 \notin B$.

13. $D \subseteq C$ since every element of D is an element of C.

14. $E \subseteq F$ since every element of E is an element of F.

15. $A \subseteq B$ and $B \subseteq A$. (Also $A = B$.)

16. $D \subseteq F$ and $F \subseteq D$. (Also $D = F$.)

17. Yes. $A \subseteq B$ and $B \subseteq A$. Thus, $A = B$.

18. $A \neq D$

19. No. $D \neq E$ because $4 \in E$ and $4 \notin D$.

20. $F = G$

21. A and B are disjoint since they have no elements in common. B and D are disjoint since they have no elements in common. C and D are disjoint.

22. $\varnothing$

23. $A \cap B = \{4, 6\}$ since 4 and 6 are elements of each set.

24. $A \cap B = \{a, d, e\}$ since a, d, and e are elements of each set.

25. $A \cap B = \varnothing$ since they have no common elements.

26. $A \cap B = \{3\}$

27. $A \cup B = \{1, 2, 3, 4, 5\}$

28. $A \cup B = \{a, b, c, d, e, i, o, u\}$

29. $A \cup B = \{1, 2, 3, 4\}$ or $A \cup B = B$.

30. $A \cup B = \{x: x \text{ is a natural number not equal to } 5\}$

For problems 31 - 42, we have
$U = \{1, 2, 3, \ldots, 9, 10\}$.

31. $A' = \{4, 6, 9, 10\}$ since these are the only elements in U that are not elements of A.

32. $B' = \{1, 2, 5, 6, 7, 9\}$
since these are the only elements in U that are not elements of B.

33. $B' = \{1, 2, 5, 6, 7, 9\}$
$A \cap B' = \{1, 2, 5, 7\}$

34. $A' = \{4, 6, 9, 10\}$
$B' = \{1, 2, 5, 6, 7, 9\}$
$A' \cap B' = \{6, 9\}$

35. $A \cup B = \{1, 2, 3, 4, 5, 7, 8, 10\}$
$(A \cup B)' = \{6, 9\}$

36. $A \cap B = \{3, 8\}$
$(A \cap B)' = \{1, 2, 4, 5, 6, 7, 9, 10\}$

37. $A' = \{4, 6, 9, 10\}$
$B' = \{1, 2, 5, 6, 7, 9\}$
$A' \cup B' = \{1, 2, 4, 5, 6, 7, 9, 10\}$

38. $A' = \{4, 6, 9, 10\}$
$B = \{3, 4, 8, 10\}$
$A' \cup B = \{3, 4, 6, 8, 9, 10\}$
$(A' \cup B)' = \{1, 2, 5, 7\}$

39. $B' = \{1, 2, 5, 6, 7, 9\}$
$C' = \{1, 3, 5, 7, 9\}$
$A \cap B' = \{1, 2, 3, 5, 7, 8\} \cap \{1, 2, 5, 6, 7, 9\}$
$= \{1, 2, 5, 7\}$
$(A \cap B') \cup C' = \{1, 2, 3, 5, 7, 9\}$

40. $A = \{1, 3, 5, 8, 7, 2\}$
$B' = \{1, 2, 5, 6, 7, 9\}$
$C' = \{1, 3, 5, 7, 9\}$
$B' \cup C' = \{1, 2, 3, 5, 6, 7, 9\}$
$A \cap (B' \cup C') = \{1, 2, 3, 5, 7\}$

41. $B' = \{1, 2, 5, 6, 7, 9\}$
$A \cap B' = \{1, 2, 3, 5, 7, 8\} \cap \{1, 2, 5, 6, 7, 9\}$
$= \{1, 2, 5, 7\}$
$(A \cap B')' \cap C = \{3, 4, 6, 8, 9, 10\} \cap \{2, 4, 6, 8, 10\}$
$= \{4, 6, 8, 10\}$

42. $B \cup C = \{2, 3, 4, 6, 8, 10\}$
$A \cap (B \cup C) = \{2, 3, 8\}$

For problems 43 - 46, we have
U = {1, 2, 3, . . . , 8, 9}.

43. $A - B = \{1, 3, 7, 9\} - \{3, 5, 8, 9\} = \{1, 7\}$

44. $A - B = \{1, 2, 3, 6, 9\} - \{1, 4, 5, 6, 7\} = \{2, 3, 9\}$

45. $A - B = \{2, 1, 5\} - \{1, 2, 3, 4, 5, 6\} = \varnothing$ or $\{\ \}$

46. $A - B = \{1, 2, 3, 4, 5\} - \{7, 8, 9\} = \{1, 2, 3, 4, 5\}$

47. a. L = {2000, 2001, 2004, 2005, 2006, 2007, 2010, 2011, 2012}
H = {2000, 2001, 2006, 2007, 2008, 2010, 2011, 2012}
C = {2001, 2002, 2003, 2008, 2009}
b. no
c. C' is the set of all years when the percentage change from low to high was 35% or less.
d. H' = {2002, 2003, 2004, 2005, 2009}
C' = {2000, 2004, 2005, 2006, 2007, 2010, 2011, 2012}
$H' \cup C'$ = {2000, 2002, 2003, 2004, 2005, 2006, 2007, 2009, 2010, 2011, 2012}. $H' \cup C'$ is the set of years when the high was less than or equal to 11,000 or the percent change was less than or equal to 35%.
e. L' = {2002, 2003, 2008, 2009}
$L' \cap C$ = {2002, 2003, 2008, 2009}.
$L' \cap C$ is the set of years when the low was less than or equal to 8,000 and the percent change was more than 35%.

48. a. A = {O, L, P}
B = {L, P}
C = {O, M, P}
b. $B \subseteq A$
c. $A \cap C = \{O, P\}$; this is the set of cities with at least 2,000,000 jobs in 2000 or 2025 and projected annual growth rates of at least 2.5%.
d. B' is the set of cities with fewer than 1,500,000 jobs in 2000.

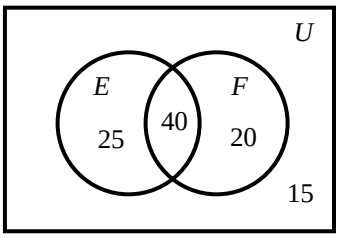
49. a. From the table, there are 100 white Republicans and 30 non-white Republicans who favor national health care, for a total of 130.
b. From the table, there are 350 + 40 Republicans, and 250 + 200 Democrats who favor national health care, for a total of 840.
c. From the table, there are 350 white Republicans, and 150 white Democrats and 20 non-whites who oppose national health care, for a total of 520.

50. **a.** From the table, 250 white Republicans and 150 white Democrats oppose national health care, for a total of 400.

b. From the table, there are 750 whites and there are 20 non-whites who oppose national health care. The total of this group is 770.

c. From the table, there are 200 non-white Democrats who favor national health care.

51. **a.** The key to solving this problem is to work from "the inside out". There are 40 aides in $E \cap F$. This leaves 65 – 40 = 25 aides who speak English but do not speak French. Also we have 60 – 40 = 20 aides who speak French but do not speak English. Thus there are 40 + 25 + 20 = 85 aides who speak English or French. This means there are 15 aides who do not speak English or French.



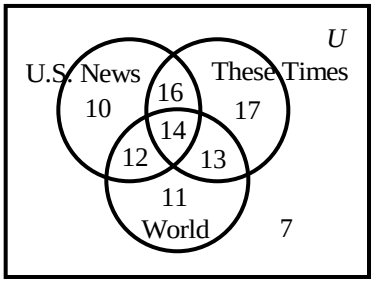
b. From the Venn diagram $E \cap F$ has 40 aides.

c. From the Venn diagram $E \cup F$ has 85 aides.

d. From the Venn diagram $E \cap F'$ has 25 aides.

52. **a.** There are 14 advertisers in the intersection of the sets. Since 30 advertised in *These Times* and *U.S. News* and we already have 14 in the center, 16 advertised in *These Times* and *U.S. News* and not in *World*. Since 26 advertised in *World* and *U.S. News* and we already have 14 in the center, 12 advertised in *World* and *U.S. News* and not in *These Times*. Since 27 advertised in *World* and *These Times* and we already have 14 in the middle, 13 advertised in *World* and *These Times* and not in *U.S. News*. 60 advertised in *These Times* and we have already accounted for 43, so 17 advertised in *These Times* only. 52 advertised in *U.S. News* and we have already accounted for 42, so 10 advertised in *U.S. News* only. 50 advertised in *World* and we have already accounted for 39, so 11 advertised in *World* only.

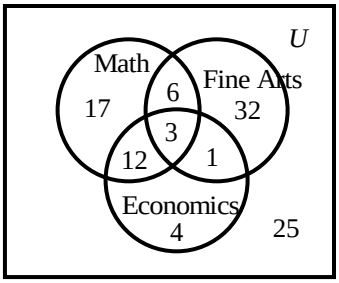
b. In the union of the 3 publications we have 10 + 16 + 17 + 14 + 12 + 13 + 11 = 93 advertisers. Thus, there are 100 – 93 = 7 who advertised in none of these publications.



c. There are 17 advertisers in the *These Times* circle that are not in an intersection.

d. In the union of *U.S. News* and *These Times* we have 10 + 12 + 16 + 14 + 17 + 13 = 82 advertisers.

53. Since 12 students take *M* and *E* but not *FA*, and 15 take *M* and *E*, 3 take all three classes. Since 9 students take *M* and *FA* and we have already counted 3, there are 6 taking *M* and *FA* which are not taking *E*. Since 4 students take *E* and *FA* and we have already counted 3, there is only 1 taking *E* and *FA* but not taking *M* also. Since 20 students take *E* and we already have 16 enrolled in *E*, this leaves 4 taking only *E*. Since 42 students take *FA* and we already have 10 enrolled in *FA*, this leaves 32 taking only *FA*. Since 38 students take *M* and we already have 21 enrolled in *M*, this leaves 17 taking only *M*.



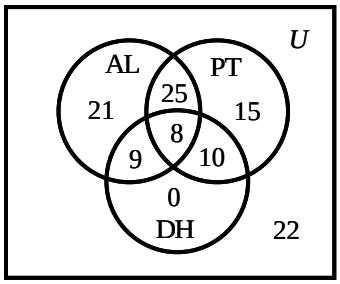
a. In the union of the 3 courses we have 17 + 12 + 3 + 6 + 32 + 1 + 4 = 75 students enrolled. Thus, there are 100 – 75 = 25 students who are not enrolled in any of these courses.

b. In $M \cup E$ we have 17 + 12 + 3 + 6 + 1 + 4 = 43 enrolled.

c. We have 17 + 32 + 4 = 53 students enrolled in exactly one of the courses.

54. Start by filling in the parts of the diagram for AL, since we have more information about it. 21 liked AL only. Since 30 liked AL but not PT, 9 liked AL or PT exclusively. 25 liked PT or AL but not DH, and 63 liked AL.

That leaves 63 – (21 + 25 + 9) = 8 in the intersection of all 3. Since 18 liked PT and DH, only 10 liked PT and DH but not AL. Since 27 liked DH, 27 – (9 + 8 + 10) = 0 liked DH only. And since 58 liked PT, 58 – (25 + 8 + 10) = 15 liked PT only.

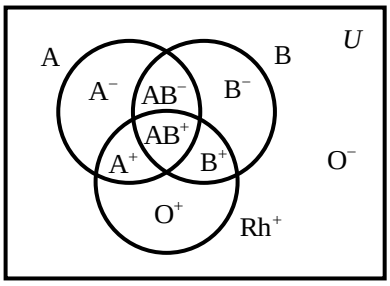


a. The number of students that liked PT or DH is 25 + 15 + 9 + 8 + 10 + 0 = 67.

b. The number that liked all three is 8.

c. The number that liked only DH is 0.

55. **a.** and **b.**



c. A^+ : 34%; B^+ : 9%; O^+ : 38%; AB^+ : 3%; O^- : 7%; A^- : 6%; B^- : 2%; AB^- : 1%

Exercises 0.2

1. **a.** Note that $-\frac{\pi}{10} = \pi \cdot \left(-\frac{1}{10}\right)$, where π is irrational and $-\frac{1}{10}$ is rational. The product of a rational number other than 0 and an irrational number is an irrational number.

b. –9 is rational and an integer.

c. $\frac{9}{3} = \frac{3}{1} = 3$. This is a natural number, an integer, and a rational number.

d. Division by zero is meaningless.

2. **a.** $\frac{0}{6} = 0$ is rational and an integer.

b. rational

c. rational

d. rational

3. **a.** Commutative

b. Distributive

c. Associative

d. Additive Identity

4. **a.** Multiplicative Identity

b. Additive Inverse

c. Multiplicative Inverse

d. Commutative

5. $-6 < 0$

6. $2 > -20$

7. $-14 < -3$

8. $\pi > 3.14$

9. $0.333 < \frac{1}{3}\left(\frac{1}{3} = 0.3333\cdots\right)$

10. $\frac{1}{3} + \frac{1}{2} = \frac{5}{6}$

11. $|-3| + |5| > |-3+5|$

12. $|-9-3| = |-9| + |3|$ (12 = 12)

13. $-3^2 + 10 \cdot 2 = -3^2 + 20 = -9 + 20 = 11$

14. $(-3)^2 + 10 \cdot 2 = 9 + 20 = 29$

15. $\dfrac{4+2^2}{2} = \dfrac{4+4}{2} = \dfrac{8}{2} = 4$

16. $\dfrac{(4+2)^2}{2} = \dfrac{6^2}{2} = \dfrac{36}{2} = 18$

17. $\dfrac{16-(-4)}{8-(-2)} = \dfrac{16+4}{8+2} = \dfrac{20}{10} = 2$

18. $\dfrac{(-5)(-3)-(-2)(3)}{-9+2} = \dfrac{15-(-6)}{-7} = \dfrac{15+6}{-7}$
$= \dfrac{21}{-7} = -3$

19. $\dfrac{|5-2|-|-7|}{|5-2|} = \dfrac{|3|-|-7|}{|3|} = \dfrac{3-7}{3} = -\dfrac{4}{3}$

20. $\dfrac{|3-|4-11||}{-|5^2-3^2|} = \dfrac{|3-|-7||}{-|25-9|}$
$= \dfrac{|3-7|}{-|16|}$
$= \dfrac{|-4|}{-16}$
$= \dfrac{4}{-16} = -\dfrac{1}{4}$

21. $\dfrac{(-3)^2-2\cdot 3+6}{4-2^2+3} = \dfrac{9-6+6}{4-4+3} = \dfrac{9}{3} = 3$

22. $\dfrac{6^2-4(-3)(-2)}{6-6^2\div 4} = \dfrac{36-(-12)(-2)}{6-36\div 4}$
$= \dfrac{36-24}{6-9}$
$= \dfrac{12}{-3}$
$= -4$

23. $\dfrac{-4^2+5-2\cdot 3}{5-4^2} = \dfrac{-16+5-6}{5-16} = \dfrac{-17}{-11} = \dfrac{17}{11}$

24. $\dfrac{3-2(5-2)}{(-2)^2-2^2+3} = \dfrac{3-2\cdot 3}{4-4+3} = \dfrac{-3}{3} = -1$

25. The entire line

26. The interval notation corresponding to $x \ge 0$ is $[0,\infty)$.

27. (1, 3]; half-open interval

28. [–4, 3]; closed interval

29. (2, 10); open interval

30. $[2,\infty)$; half-open interval

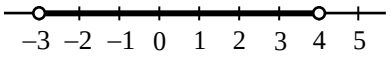
31. $-3 \le x < 5$

32. $x > -2$

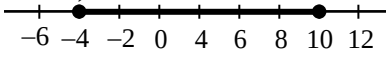
33. $x > 4$

34. $0 \le x < 5$

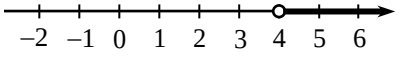
35. $(-\infty, 4) \cap (-3, \infty) = (-3, 4)$



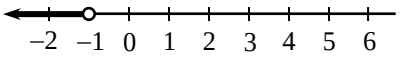
36. $[-4, 17) \cap [-20, 10] = [-4, 10]$



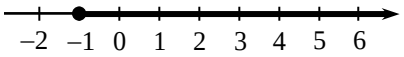
37. $x > 4$ and $x \ge 0 = (4,\infty)$



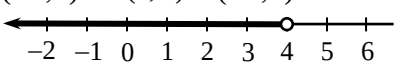
38. $x < 10$ and $x < -1$ is $x < -1$ or $(-\infty,-1)$.



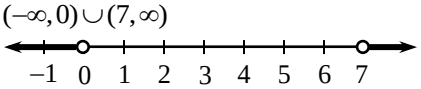
39. $[0, \infty) \cup [-1, 5] = [-1,\infty)$



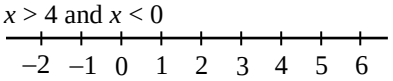
40. $(-\infty,4) \cup (0, 2) = (-\infty,4)$



41. $(-\infty,0) \cup (7,\infty)$



42. $x > 4$ and $x < 0$



The intersection is the empty set.

43. –0.000038585

44. 0.404787025

45. 9122.387471

46. 11.80591621

47. $\dfrac{2500}{[(1.1^6)-1]} = \dfrac{2500}{0.771561} = 3240.184509$

48. 1591.712652

49. **a.** $\$300.00 + \$788.91 = \$1088.91$

b. Federal withholding
= 0.25(1088.91 – 54.45) = $258.62

c. Retirement: 0.05(1088.91) = $54.45
State tax = Retirement = $54.45
Local tax = 0.01(1088.91) = $10.89
Federal tax = $258.62 (from **b.** above)
Social Security and Medicare tax
= 0.0765(1088.91) = $83.30
Total Withholding = $461.71
Take-home = 1088.91 – 461.71 = $627.20

50. **a.** $t = 2010 - 2000 = 10$

b. $E = 5.03(10)^2 + 100(10) + 1380$
$= \$2883$ billion

c. $t = 2015 - 2000 = 15$
$E = 50.3(15)^2 + 100(15) + 1380$
$= \$4011.75$ billion

51. **a.** Equation (1) is more accurate.
Equation (1) gives
$y = 0.207(13) - 0.000370$
= 2.69 billion

Equation (2) gives
$y = 0.00454(13)^2 + 0.126(13) + 0.271$
= 2.68 billion

b. For 2018, Equation (1) gives
$y = 0.207(18) - 0.000370$
= 3.73 billion

For 2018, Equation (2) gives
$y = 0.00454(18)^2 + 0.126(18) + 0.271$
= 4.01 billion

52. **a.** $H = 2.31(10.5) + 31.26 = 55.515$ inches
Upper: $1.05(55.515) = 58.29$ inches
Lower: $0.95(55.515) = 52.74$ inches
$52.74 \le H \le 58.29$

b. $H = 2.31(5.75) + 31.26 = 44.5425$ inches
Upper: $1.05(44.5425) = 46.77$ inches
Lower: $0.95(44.5425) = 42.32$ inches
$42.32 \le H \le 46.77$

53. **a.** $\$82,401 \le I \le 171,850$;
$\$171,851 \le I \le \$373,650$;
$I > \$373,650$

b. $T = \$4681.25$ for $I = \$34,000$
$T = \$16,781.25$ for $I = \$82,400$

c. $[4681.25,\ 16,781.25]$

Exercises 0.3

1. $(-4)^4 = (-4)(-4)(-4)(-4) = 256$

2. $-5^3 = -1 \cdot 5 \cdot 5 \cdot 5 = -125$

3. $-2^6 = -1 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = -64$

4. $(-2)^5 = (-2)(-2)(-2)(-2)(-2) = -32$

5. $3^{-2} = \dfrac{1}{3^2} = \dfrac{1}{9}$

6. $6^{-1} = \dfrac{1}{6}$

7. $-\left(\dfrac{3}{2}\right)^2 = (-1)\left(\dfrac{3}{2}\right)\left(\dfrac{3}{2}\right) = -\dfrac{9}{4}$

8. $\left(\dfrac{2}{3}\right)^3 = \dfrac{2^3}{3^3} = \dfrac{8}{27}$

9. 1.2 [y^x] 4 [=] 2.0736

10. –3.7 [y^x] 3 [=] –50.653

11. 1.5 [y^x] –5 [=] 0.1316872428

12. –0.8 [y^x] –9 [=] –7.450580597

13. $6^5 \cdot 6^3 = 6^{5+3} = 6^8$

14. $8^4 \cdot 8^2 \cdot 8 = 8^{4+2+1} = 8^7$

15. $\dfrac{10^8}{10^9} = 10^{8-9} = 10^{-1} = \dfrac{1}{10}$

16. $\dfrac{7^8}{7^3} = 7^{8-3} = 7^5$

17. $\dfrac{9^4 \cdot 9^{-7}}{9^{-3}} = \dfrac{9^{4+(-7)}}{9^{-3}} = \dfrac{9^{-3}}{9^{-3}} = 9^{-3-(-3)} = 9^0 = 1$

18. $\dfrac{5^4}{\left(5^{-2} \cdot 5^3\right)} = \dfrac{5^4}{5^{-2+3}} = \dfrac{5^4}{5^1} = 5^{4-1} = 5^3$

19. $\left(3^3\right)^3 = 3^{3 \cdot 3} = 3^9$

20. $\left(2^{-3}\right)^{-2} = 2^{(-3)\cdot(-2)} = 2^6$

21. $\left(\dfrac{2}{3}\right)^{-2} = \left(\dfrac{3}{2}\right)^2 = \dfrac{9}{4}$

22. $\left(\dfrac{-2}{5}\right)^{-4} = \left(\dfrac{5}{-2}\right)^4 = \left(-\dfrac{5}{2}\right)^4$

23. $-x^{-3} = -1 \cdot x^{-3} = -1 \cdot \dfrac{1}{x^3} = -\dfrac{1}{x^3}$

24. $x^{-4} = \dfrac{1}{x^4}$

25. $xy^{-2}z^0 = x \cdot \dfrac{1}{y^2} \cdot 1 = \dfrac{x}{y^2}$

26. $4^{-1}x^0y^{-2} = \dfrac{1}{4} \cdot 1 \cdot \dfrac{1}{y^2} = \dfrac{1}{4y^2}$

27. $x^3 \cdot x^4 = x^{3+4} = x^7$

28. $a^5 \cdot a = a^{5+1} = a^6$

29. $x^{-5} \cdot x^3 = x^{-5+3} = x^{-2} = \dfrac{1}{x^2}$

30. $y^{-5} \cdot y^{-2} = y^{-5+(-2)} = y^{-7} = \dfrac{1}{y^7}$

31. $\dfrac{x^8}{x^4} = x^{8-4} = x^4$

32. $\dfrac{a^5}{a^{-1}} = a^{5-(-1)} = a^{5+1} = a^6$

33. $\dfrac{y^5}{y^{-7}} = y^{5-(-7)} = y^{12}$

34. $\dfrac{y^{-3}}{y^{-4}} = y^{-3-(-4)} = y^{-3+4} = y^1 = y$

35. $\left(x^4\right)^3 = x^{3 \cdot 4} = x^{12}$

36. $(y^3)^{-2} = y^{3\cdot(-2)} = y^{-6} = \dfrac{1}{y^6}$

37. $(xy)^2 = x^2y^2$

38. $(2m)^3 = 2^3m^3 = 8m^3$

39. $\left(\dfrac{2}{x^5}\right)^4 = \dfrac{2^4}{\left(x^5\right)^4} = \dfrac{16}{x^{5 \cdot 4}} = \dfrac{16}{x^{20}}$

40. $\left(\dfrac{8}{a^3}\right)^3 = \dfrac{8^3}{(a^3)^3} = \dfrac{512}{a^{3 \cdot 3}} = \dfrac{512}{a^9}$

41. $\left(2x^{-2}y\right)^{-4} = 2^{-4}x^8y^{-4} = \dfrac{x^8}{16y^4}$

42. $\left(-32x^5\right)^{-3} = (-32)^{-3}\left(x^5\right)^{-3}$

$= \dfrac{1}{(-32)^3}x^{5(-3)}$

$= \dfrac{1}{-32768} \cdot x^{-15}$

$= -\dfrac{1}{32768x^{15}}$

43. $\left(-8a^{-3}b^{2}\right)\left(2a^{5}b^{-4}\right) = -16a^{-3+5}b^{2-4}$

$= -16a^{2}b^{-2}$

$= -\dfrac{16a^{2}}{b^{2}}$

44. $\left(-3m^{2}y^{-1}\right)\left(2m^{-3}y^{-1}\right) = -6m^{2+(-3)}y^{-1+(-1)}$

$= -6m^{-1}y^{-2}$

$= -6\left(\dfrac{1}{m}\right)\left(\dfrac{1}{y^{2}}\right)$

$= \dfrac{-6}{my^{2}}$

45. $\left(2x^{-2}\right) \div \left(x^{-1}y^{2}\right) = \dfrac{2}{x^{2}} \div \dfrac{y^{2}}{x} = \dfrac{2}{x^{2}} \cdot \dfrac{x}{y^{2}} = \dfrac{2}{xy^{2}}$

46. $\left(-8a^{-3}b^{2}c\right) \div \left(2a^{5}b^{4}\right) = \dfrac{-8a^{-3}b^{2}c}{2a^{5}b^{4}}$

$= \dfrac{-8}{2} \cdot \dfrac{a^{-3}}{a^{5}} \cdot \dfrac{b^{2}}{b^{4}} \cdot c$

$= \dfrac{-4c}{a^{8}b^{2}}$

47. $\left(\dfrac{x^{3}}{y^{-2}}\right)^{-3} = \dfrac{x^{-9}}{y^{6}} = \dfrac{1}{x^{9}} \cdot \dfrac{1}{y^{6}} = \dfrac{1}{x^{9}y^{6}}$

48. $\left(\dfrac{x^{-2}}{y}\right)^{-3} = \dfrac{(x^{-2})^{-3}}{y^{-3}} = \dfrac{x^{(-2)(-3)}}{y^{-3}} = \dfrac{x^{6}}{y^{-3}} = x^{6} \cdot \dfrac{y^{3}}{1}$

$= x^{6}y^{3}$

49. $\left(\dfrac{a^{-2}b^{-1}c^{-4}}{a^{4}b^{-3}c^{0}}\right)^{-3} = \left(\dfrac{b^{2}}{a^{6}c^{4}}\right)^{-3} = \left(\dfrac{a^{6}c^{4}}{b^{2}}\right)^{3} = \dfrac{a^{18}c^{12}}{b^{6}}$

50. $\left(\dfrac{4x^{-1}y^{-40}}{2^{-2}x^{4}y^{-10}}\right)^{-2} = \left(\dfrac{4}{(1/2)^{2}} \cdot x^{-1-4} \cdot y^{-40-(-10)}\right)^{-2}$

$= (16x^{-5}y^{-30})^{-2}$

$= (16)^{-2}(x^{-5})^{-2}(y^{-30})^{-2}$

$= \dfrac{1}{(16)^{2}} \cdot x^{(-5)(-2)}y^{(-30)(-2)}$

$= \dfrac{1}{256}x^{10}y^{60}$

$= \dfrac{x^{10}y^{60}}{256}$

51. a. $\dfrac{2x^{-2}}{(2x)^{2}} = 2 \cdot \dfrac{1}{x^{2}} \cdot \dfrac{1}{(2x)^{2}} = 2 \cdot \dfrac{1}{x^{2}} \cdot \dfrac{1}{4x^{2}} = \dfrac{1}{2x^{4}}$

b. $\dfrac{(2x)^{-2}}{(2x)^{2}} = \dfrac{1}{(2x)^{2}} \cdot \dfrac{1}{(2x)^{2}} = \dfrac{1}{4x^{2}} \cdot \dfrac{1}{4x^{2}} = \dfrac{1}{16x^{4}}$

c. $\dfrac{2x^{-2}}{2x^{2}} = 2 \cdot \dfrac{1}{x^{2}} \cdot \dfrac{1}{2x^{2}} = \dfrac{1}{x^{4}}$

d. $\dfrac{2x^{-2}}{(2x)^{-2}} = 2 \cdot \dfrac{1}{x^{2}} \cdot (2x)^{2} = 2 \cdot \dfrac{1}{x^{2}} \cdot 4x^{2} = 8$

52. a. $\dfrac{2^{-1}x^{-2}}{(2x)^{2}} = \dfrac{2^{-1}x^{-2}}{2^{2}x^{2}} = 2^{-1-2}x^{-2-2}$

$= 2^{-3}x^{-4} = \dfrac{1}{8x^{4}}$

b. $\dfrac{2^{-1}x^{-2}}{2x^{2}} = 2^{-1-1}x^{-2-2} = 2^{-2}x^{-4} = \dfrac{1}{4x^{4}}$

c. $\dfrac{\left(2x^{-2}\right)^{-1}}{(2x)^{-2}} = \dfrac{2^{-1}x^{(-2)(-1)}}{2^{-2}x^{-2}} = \dfrac{2^{-1}x^{2}}{2^{-2}x^{-2}}$

$= 2^{-1-(-2)}x^{2-(-2)} = 2x^{4}$

d. $\dfrac{(2x^{-2})^{-1}}{2x^{2}} = \dfrac{2^{-1}x^{(-2)(-1)}}{2x^{2}} = \dfrac{2^{-1}x^{2}}{2x^{2}}$

$= 2^{-1-1}x^{2-2} = \dfrac{1}{4}$

53. $\dfrac{1}{x} = x^{-1}$

54. $\dfrac{1}{x^{2}} = x^{-2}$

55. $(2x)^{3} = 2^{3}x^{3} = 8x^{3}$

56. $(3x)^{2} = 3^{2}x^{2} = 9x^{2}$

57. $\dfrac{1}{(4x^2)} = \dfrac{1}{4}\cdot\dfrac{1}{x^2} = \dfrac{1}{4}x^{-2}$

58. $\dfrac{3}{(2x^4)} = \dfrac{3}{2}\cdot\dfrac{1}{x^4} = \dfrac{3}{2}x^{-4}$

59. $\left(\dfrac{-x}{2}\right)^3 = \dfrac{-x^3}{2^3} = -\dfrac{1}{8}x^3$

60. $\left(\dfrac{-x}{3}\right)^2 = \dfrac{(-x)^2}{3^2} = \dfrac{x^2}{9} = \dfrac{1}{9}x^2$

61. $P = 1200,\ i = 0.12,\ n = 5$

$S = P(1+i)^n$

$= 1200(1+0.12)^5$

$= 1200(1.12)^5$

$= \$2114.81$

$I = S - P = 2114.81 - 1200 = \$914.81$

62. $P = 1800,\ i = 0.10,\ n = 7$

$S = P(1+i)^n$

$= 1800(1+0.10)^7$

$= 1800(1.10)^7$

$= 1800(1.9487171)$

$= \$3507.69$

$I = S - P = 3507.69 - 1800 = \$1707.69$

63. $P = 5000,\ i = 0.115,\ n = 6$

$S = P(1+i)^n$

$= 5000(1+0.115)^6$

$= 5000(1.115)^6$

$= \$9607.70$

$I = S - P = 9607.70 - 5000 = \$4607.70$

64. $P = 800,\ i = 0.105,\ n = 20$

$S = P(1+i)^n$

$= 800(1+0.105)^{20}$

$= 800(1.105)^{20}$

$= 5892.99$

$I = S - P = 5892.99 - 800 = \$5092.99$

65. $S = 15{,}000,\ n = 6,\ i = 0.115$

$P = S(1+i)^{-n}$

$= 15{,}000(1+0.115)^{-6}$

$= 15{,}000(1.115)^{-6}$

$= \$7806.24$

66. $S = 80{,}000,\ n = 20,\ i = 0.105$

$P = S(1+i)^{-n}$

$= 80{,}000(1+0.105)^{-20}$

$= 80{,}000(1.105)^{-20}$

$= \$10{,}860.37$

67. $I = 492.4(1.070)^t$

a.

Year	1980	2000	2008
t-value	20	40	48

b.

Income (in billions)	\$1905	\$7373	\$12,669

c. $I = 492.4(1.070)^{58} \approx \$24{,}922$ billion

68. **a.** $t = 2019 - 2010 = 9$

b. $y = 0.012(1.75)^9 \approx 1.8$ billion cubic feet

c. $y = 0.012(1.75)^{12} \approx 9.9$ billion cubic feet

69. $y = \dfrac{1095}{1+10.12(1.212)^{-t}}$

a.

Year	1990	2003	2012
t – value	10	23	32
Predicted number of endangered species	442	976	1072

b. Year 2020: $t = 40$; $y = \dfrac{1095}{1+10.12(1.212)^{-40}} \approx 1090$

Increase between 2007 and 2020 is $1090 - 1037 = 53$ species

c. Two possibilities might be more environmental protections and the fact that there are only a limited number of species.

d. There are only a limited number of species. Also, below some threshold level the ecological balance might be lost, perhaps resulting in an environmental catastrophe (which the equation could not predict). To find the upper limit, which is 1095, compute y for large t-values:

Year	2040	2100	2200
t – value	60	120	220
Predicted number of endangered species	1094.9	1095	1095

70. $p = \dfrac{249.6}{1+1.915(1.028)^{-t}}$

a.

Year	1980	2000	2020
t – value	30	50	70
U.S. population (age 20 - 64) in millions	135.92	168.49	195.44

b. Year 2025: $t = 75$; $p = \dfrac{249.6}{1+1.915(1.028)^{-75}} \approx 201.07$ million

Year 2045: $t = 95$; $p = \dfrac{249.6}{1+1.915(1.028)^{-95}} \approx 219.15$ million

The increase from 2025 and 2045 is predicted to be $219.15 - 201.07 = 18.08$ million. This is less than the predicted 28.4 million increase from 2000 to 2020.

c. It is reasonable for a formula such as this to have an upper limit that cannot be exceeded because there are limited resources and space. To find the upper limit, which is 249.6 million, compute p for large t-values:

Year	2150	2350	2450
t – value	200	400	500
U.S. population (age 20 - 64) in millions	247.7	249.6	249.6

71. $H = 738.1(1.065)^t$

a. $t = 10$ corresponds to 2000.

b. $H = 738.1(1.065)^{10} \approx \$1385.5$ billion

c. $H = 738.1(1.065)^{20} \approx \$2600.8$ billion

d. $H = 738.1(1.065)^{28} \approx \$4304.3$ billion

Exercises 0.4

1. **a.** Since $\left(\frac{16}{3}\right)^2 = \frac{256}{9}$ we have

$$\sqrt{\frac{256}{9}} = \frac{16}{3} \approx 5.33$$

b. $\sqrt{1.44} = 1.2$

2. **a.** $\sqrt[5]{-32^3} = \sqrt[5]{-1 \cdot 32^3} = -\sqrt[5]{32^3} = -(32)^{3/5}$

$$= -\left(\sqrt[5]{32}\right)^3 = -(2)^3 = -8$$

b. $\sqrt[4]{-16^5} = \sqrt[4]{-1048576}$ The square root of a negative number is not real.

3. **a.** $16^{3/4} = \left(\sqrt[4]{16}\right)^3 = 2^3 = 8$

b. $(-16)^{-3/2} = \left(\sqrt{-16}\right)^{-3}$ The square root of a negative number is not real.

4. **a.** $-27^{-1/3} = -\left(27^{-1/3}\right) = -\frac{1}{\sqrt[3]{27}} = -\frac{1}{3}$

b. $32^{3/5} = \left(\sqrt[5]{32}\right)^3 = 2^3 = 8$

5. $\left(\frac{8}{27}\right)^{-2/3} = \left(\frac{27}{8}\right)^{2/3} = \left(\sqrt[3]{\frac{27}{8}}\right)^2 = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$

6. $\left(\frac{4}{9}\right)^{3/2} = \left(\sqrt{\frac{4}{9}}\right)^3 = \left(\frac{2}{3}\right)^3 = \frac{8}{27}$

7. **a.** $64^{2/3} = \left(\sqrt[3]{64}\right)^2 = 4^2 = 16$

b. $(-64)^{-2/3} = \frac{1}{(-64)^{2/3}} = \frac{1}{\left(\sqrt[3]{-64}\right)^2}$

$$= \frac{1}{(-4)^2} = \frac{1}{16}$$

8. **a.** $64^{-2/3} = \frac{1}{64^{2/3}} = \frac{1}{\left(\sqrt[3]{64}\right)^2} = \frac{1}{4^2} = \frac{1}{16}$

b. $-64^{2/3} = -\left(\sqrt[3]{64}\right)^2 = -(4)^2 = -16$

9. $\sqrt[9]{(6.12)^4} = (6.12)^{4/9} \approx 2.237$

10. $\sqrt[12]{4.96} = (4.96)^{1/12} \approx 1.1428$

11. $\sqrt{m^3} = m^{3/2}$

12. $\sqrt[3]{x^5} = x^{5/3}$

13. $\sqrt[4]{m^2 n^5} = \left(m^2 n^5\right)^{1/4} = m^{2/4} n^{5/4} = m^{1/2} n^{5/4}$

14. $\sqrt[5]{x^3} = x^{3/5}$

15. $2x^{\frac{1}{2}} = 2\sqrt{x}$

16. $12x^{\frac{1}{4}} = 12\sqrt[4]{x}$

17. $x^{7/6} = \sqrt[6]{x^7}$

18. $y^{11/5} = \sqrt[5]{y^{11}}$

19. $-\left(\frac{1}{4}\right)x^{-5/4} = -\frac{1}{4} \cdot \frac{1}{x^{5/4}} = \frac{-1}{4\sqrt[4]{x^5}}$

20. $-x^{-5/3} = -\sqrt[3]{x^{-5}} = \frac{-1}{\sqrt[3]{x^5}}$

21. $y^{1/4} \cdot y^{1/2} = y^{(1/4)+(1/2)} = y^{3/4}$

22. $x^{2/3} \cdot x^{1/5} = x^{(2/3)+(1/5)} = x^{(10/15)+(3/15)} = x^{13/15}$

23. $z^{3/4} \cdot z^4 = z^{(3/4)+(16/4)} = z^{19/4}$

24. $x^{-2/3} \cdot x^2 = x^{(-2/3)+2} = x^{(-2/3)+(6/3)} = x^{4/3}$

25. $y^{-3/2} \cdot y^{-1} = y^{(-3/2)-(2/2)} = y^{-5/2} = \dfrac{1}{y^{5/2}}$

26. $z^{-2} \cdot z^{5/3} = z^{-2+(5/3)} = z^{(-6/3)+(5/3)} = z^{-1/3} = \dfrac{1}{z^{1/3}}$

27. $\dfrac{x^{\frac{1}{3}}}{x^{\frac{-2}{3}}} = x^{\left(\frac{1}{3}\right)-\left(\frac{-2}{3}\right)} = x^{\frac{3}{3}} = x$

28. $\dfrac{x^{-1/2}}{x^{-3/2}} = x^{(-1/2)-(-3/2)} = x^{(-1/2)+(3/2)} = x^{2/2} = x$

29. $\dfrac{y^{-5/2}}{y^{-2/5}} = y^{(-5/2)-(-2/5)} = y^{(-25/10)+(4/10)}$

$= y^{-21/10} = \dfrac{1}{y^{21/10}}$

30. $\dfrac{x^{4/9}}{x^{1/12}} = x^{(4/9)-(1/12)} = x^{(16/36)-(3/36)} = x^{13/36}$

31. $(x^{2/3})^{3/4} = x^{(2/3)(3/4)} = x^{2/4} = x^{1/2}$

32. $(x^{4/5})^3 = x^{(4/5)(3)} = x^{12/5}$

33. $(x^{-1/2})^2 = x^{-1} = \dfrac{1}{x}$

34. $(x^{-2/3})^{-2/5} = x^{(-2/3)(-2/5)} = x^{4/15}$

35. $\sqrt{64x^4} = 8x^2$

36. $\sqrt[3]{-64x^6y^3} = \sqrt[3]{-64} \cdot \sqrt[3]{x^6} \cdot \sqrt[3]{y^3} = -4x^2y$

37. $\sqrt{128x^4y^5} = \sqrt{64x^4y^4 \cdot 2y}$

$= \sqrt{64} \cdot \sqrt{x^4} \cdot \sqrt{y^4} \cdot \sqrt{2y} = 8x^2y^2\sqrt{2y}$

38. $\sqrt[3]{54x^5z^8} = \sqrt[3]{54} \cdot \sqrt[3]{x^5} \cdot \sqrt[3]{z^8}$

$= 3\sqrt[3]{2} \cdot x\sqrt[3]{x^2} \cdot z^2\sqrt[3]{z^2} = 3xz^2\sqrt[3]{2x^2z^2}$

39. $\sqrt[3]{40x^8y^5} = \sqrt[3]{8x^6y^3 \cdot 5x^2y^2}$

$= \sqrt[3]{8} \cdot \sqrt[3]{x^6} \cdot \sqrt[3]{y^3} \cdot \sqrt[3]{5x^2y^2}$

$= 2x^2y\sqrt[3]{5x^2y^2}$

40. $\sqrt{32x^5y} = \sqrt{32} \cdot \sqrt{x^5} \cdot \sqrt{y} = 4\sqrt{2} \cdot x^2\sqrt{x} \cdot \sqrt{y}$

$= 4x^2\sqrt{2xy}$

41. $\sqrt{12x^3y} \cdot \sqrt{3x^2y} = \sqrt{36x^5y^2} = \sqrt{36} \cdot \sqrt{x^5} \cdot \sqrt{y^2}$

$= 6x^2y\sqrt{x}$

42. $\sqrt[3]{16x^2y} \cdot \sqrt[3]{3x^2y} = \sqrt[3]{48x^4y^2} = \sqrt[3]{48} \cdot \sqrt[3]{x^4} \cdot \sqrt[3]{y^2}$

$= 2\sqrt[3]{6} \cdot x\sqrt[3]{x} \cdot \sqrt[3]{y^2} = 2x\sqrt[3]{6xy^2}$

43. $\sqrt{63x^5y^3} \cdot \sqrt{28x^2y} = \sqrt{9x^4y^2 \cdot 7xy} \cdot \sqrt{4x^2 \cdot 7y}$

$= 3x^2y\sqrt{7xy} \cdot 2x\sqrt{7y}$

$= 42x^3y^2\sqrt{x}$

44. $\sqrt{10xz^{10}} \cdot \sqrt{30x^{17}z} = \sqrt{300x^{18}z^{11}}$

$= \sqrt{300} \cdot \sqrt{x^{18}} \cdot \sqrt{z^{11}}$

$= 10\sqrt{3} \cdot x^9 \cdot z^5\sqrt{z}$

$= 10x^9z^5\sqrt{3z}$

45. $\dfrac{\sqrt{12x^3y^{12}}}{\sqrt{27xy^2}} = \sqrt{\dfrac{4x^2y^{10}}{9}} = \dfrac{2xy^5}{3}$

46. $\dfrac{\sqrt{250xy^7z^4}}{\sqrt{18x^{17}y^2}} = \sqrt{\dfrac{250xy^7z^4}{18x^{17}y^2}} = \sqrt{\dfrac{125y^5z^4}{9x^{16}}}$

$= \dfrac{\sqrt{125}\sqrt{y^5}\sqrt{z^4}}{\sqrt{9}\sqrt{x^{16}}}$

$= \dfrac{5\sqrt{5} \cdot y^2\sqrt{y} \cdot z^2}{3x^8}$

$= \dfrac{5y^2z^2\sqrt{5y}}{3x^8}$

47. $\dfrac{\sqrt[4]{32a^9b^5}}{\sqrt[4]{162a^{17}}} = \sqrt[4]{\dfrac{16b^4}{81a^8} \cdot \dfrac{b}{1}} = \dfrac{2b}{3a^2}\sqrt[4]{b}$

48. $\dfrac{\sqrt[3]{-16x^3y^4}}{\sqrt[3]{128y^2}} = \sqrt[3]{\dfrac{-16x^3y^4}{128y^2}} = \dfrac{\sqrt[3]{x^3}\sqrt[3]{y^2}}{\sqrt[3]{-8}}$

$= \dfrac{x\sqrt[3]{y^2}}{-2} = \dfrac{-x\sqrt[3]{y^2}}{2}$

49. $(A^9)^x = A^{9x}$

$A^{9x} = A^1$

$9x = 1$

$x = \dfrac{1}{9}$

50. $(B^{20})^x = B$

$B^{20x} = B^1$

$20x = 1$

$x = \dfrac{1}{20}$

51. $\left(\sqrt[7]{R}\right)^x = R^{x/7}$

$R^{x/7} = R^1$

$\dfrac{x}{7} = 1$

$x = 7$

52. $\left(\sqrt{T^3}\right)^x = T$

$((T^3)^{1/2})^x = T^1$

$(T^{3/2})^x = T^1$

$T^{3x/2} = T^1$

$\dfrac{3x}{2} = 1$

$x = \dfrac{2}{3}$

53. $\sqrt{\dfrac{2}{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{\sqrt{2}\cdot\sqrt{3}}{\sqrt{3}\cdot\sqrt{3}} = \dfrac{\sqrt{6}}{3}$

54. $\sqrt{\dfrac{5}{8}} \cdot \dfrac{\sqrt{8}}{\sqrt{8}} = \dfrac{\sqrt{5}}{\sqrt{8}} \cdot \dfrac{\sqrt{8}}{\sqrt{8}} = \dfrac{\sqrt{40}}{\sqrt{64}} = \dfrac{2\sqrt{10}}{8} = \dfrac{\sqrt{10}}{4}$

55. $\dfrac{\sqrt{m^2x}}{\sqrt{mx^2}} = \dfrac{\sqrt{m}}{\sqrt{x}} = \dfrac{\sqrt{m}\cdot\sqrt{x}}{\sqrt{x}\cdot\sqrt{x}} = \dfrac{\sqrt{mx}}{x}$

56. $\dfrac{5x^3w}{\sqrt{4xw^2}} \cdot \dfrac{\sqrt{x}}{\sqrt{x}} = \dfrac{5x^3w\sqrt{x}}{\sqrt{4x^2w^2}} = \dfrac{5x^3w\sqrt{x}}{2xw} = \dfrac{5x^2\sqrt{x}}{2}$

57. $\dfrac{\sqrt[3]{m^2x}}{\sqrt[3]{mx^5}} = \dfrac{\sqrt[3]{m}}{\sqrt[3]{x^4}} = \dfrac{\sqrt[3]{m}}{\sqrt[3]{x^3}\cdot\sqrt[3]{x}} \cdot \dfrac{\sqrt[3]{x^2}}{\sqrt[3]{x^2}} = \dfrac{\sqrt[3]{mx^2}}{x\sqrt[3]{x^3}}$

$= \dfrac{\sqrt[3]{mx^2}}{x^2}$

58. $\dfrac{\sqrt[4]{mx^3}}{\sqrt[4]{y^2z^5}} \cdot \dfrac{\sqrt[4]{y^2z^3}}{\sqrt[4]{y^2z^3}} = \dfrac{\sqrt[4]{mx^3y^2z^3}}{\sqrt[4]{y^4z^8}} = \dfrac{\sqrt[4]{mx^3y^2z^3}}{yz^2}$

59. $\dfrac{-2}{3\sqrt[3]{x^2}} = \dfrac{-2}{3} \cdot \dfrac{1}{x^{2/3}} = -\dfrac{2}{3}x^{-2/3}$

60. $\dfrac{-2}{3\sqrt[4]{x^3}} = \dfrac{-2}{3x^{3/4}} = \dfrac{-2x^{-3/4}}{3} = -\dfrac{2}{3}x^{-3/4}$

61. $3x\sqrt{x} = 3x \cdot x^{1/2} = 3x^{3/2}$

62. $\sqrt{x} \cdot \sqrt[3]{x} = x^{1/2}x^{1/3} = x^{(1/2)+(1/3)} = x^{(3/6)+(2/6)}$

$= x^{5/6}$

63. $\dfrac{3}{2}x^{1/2} = \dfrac{3}{2}\sqrt{x}$

64. $\dfrac{4}{3}x^{1/3} = \dfrac{4}{3}\sqrt[3]{x} = \dfrac{4\sqrt[3]{x}}{3}$

65. $\dfrac{1}{2}x^{-1/2} = \dfrac{1}{2} \cdot \dfrac{1}{x^{1/2}} = \dfrac{1}{2\sqrt{x}}$

66. $\dfrac{-1}{2}x^{-3/2} = \dfrac{-1}{2x^{3/2}} = \dfrac{-1}{2\sqrt{x^3}}$

67. **a.** $R = 8.5 = \dfrac{17}{2} \qquad I = 10^{17/2} = \sqrt{10^{17}}$

b. $I = 10^{9.0} = 1{,}000{,}000{,}000$

c. $\dfrac{I_{2011}}{I_{1989}} = \dfrac{10^{9.0}}{10^{6.9}} = 10^{2.1} \approx 125.9$

68. **a.** $10^{D/10} = (10^D)^{1/10} = \sqrt[10]{10^D}$

b. $I_1 = \sqrt[10]{10^{32}} \approx 1584.89$

c. $\dfrac{I_2}{I_1} = \dfrac{\sqrt[10]{10^{140}}}{\sqrt[10]{10^{32}}} = \sqrt[10]{10^{140-32}} = \sqrt[10]{10^{108}}$

$= 10^{(108)(1/10)} = 10^{10.8}$

$\approx 6.31 \times 10^{10}$

69. a. $S = 1000\sqrt{\left(1+\dfrac{r}{100}\right)^5}$

b. $S = 1000\sqrt{\left(1+\dfrac{6.6}{100}\right)^5} \approx \$1173.26$

70. a. $0.21 = \dfrac{21}{100}$, so $L = 29\sqrt[100]{x^{21}}$

b. $L = 29(115)^{0.21} \approx 78.5$ years

71. a. $P = 0.924t^{13/100} = 0.924\sqrt[100]{t^{13}}$

b.

Year	t	Population
2005	5	1.1390
2010	10	1.2464
2045	45	1.5156
2050	50	1.5365

Change from 2005 to 2010: 0.1074 billion

Change from 2045 to 2050: 0.0209 billion

By 2045 and 2050 the population is much larger than earlier in the 21st century, and there is a limited number of people that any land can support—in terms of both space and food.

72. a. $p = 14.32t^{0.38} = 14.32t^{38/100} = 14.32t^{19/50} = 14.32\sqrt[50]{t^{19}}$

b.

Year	t	Percent of Roads Paved
1970	20	44.7
1980	30	52.1
2000	50	63.3
2010	60	67.9

Change from 1970 to 1980: 7.4%

Change from 2000 to 2010: 4.6%

The equation estimates a greater percent change from 1970 to 1980 than from 2000 to 2010. There were fewer roads left to be paved from 2000 to 2010.

c. When a t-value makes $p > 100\%$, you can be certain that the equation is no longer valid since you cannot pave more than 100% of the roads.

73. $k = 25, t = 10, q_0 = 98$

$q = q_0(2^{-t/k})$

$= 98(2^{-10/25})$

$= 98(2^{-2/5}) \approx 74$ kg

74. $k = 5600, t = 10{,}000, q_0 = 40$

$q = q_0(2^{-t/k})$

$= 40(2^{-10{,}000/5600})$

$= 40(2^{-25/14}) \approx 11.6$ g

75. $P = P_0(2.5)^{ht} = 30{,}000(2.5)^{0.03(10)}$

$= 30{,}000(2.5)^{0.3} \approx 39{,}491$

76. $x = 10$

$$\begin{aligned} S &= 2000\left(2^{-0.1x}\right) \\ &= 2000\left(2^{-0.1(10)}\right) \\ &= 2000\left(2^{-1}\right) \\ &= 2000 \cdot \frac{1}{2} \\ &= \$1000 \end{aligned}$$

77. a. $N = 500(0.02)^{(0.7)^t}$; at $t = 0$ we have $(0.7)^0 = 1$. Thus, $N = 500(0.02)^1 = 10.$

b. $N = 500\left(0.02\right)^{(0.7)^5}$

$$\begin{aligned} &= 500\left(0.02\right)^{0.16807} \\ &= 259 \end{aligned}$$

Exercises 0.5

1. $10 - 3x - x^2$

a. The largest exponent is 2. The degree of the polynomial is 2.

b. The coefficient of x^2 is –1.

c. The constant term is 10.

d. It is a polynomial of one variable *x*.

2. $5x^4 - 2x^9 + 7$

a. The largest exponent is 9. The degree of the polynomial is 9.

b. The coefficient of x^9 is –2.

c. The constant term is 7.

d. It is a polynomial of one variable *x*.

3. $7x^2y - 14xy^3z$

a. The sum of the exponents in each term is 3 and 5, respectively. The degree of the polynomial is 5.

b. The coefficient of xy^3z is –14.

c. The constant term is zero.

d. It is a polynomial of several (three) variables: *x*, *y*, and *z*.

4. $2x^5 + 7x^2y^3 - 5y^6$

a. The sum of the exponents of each term is 5, 5 and 6, respectively. The degree of the polynomial is 6.

b. The coefficient of y^6 is –5.

c. The constant term is 0.

d. It is a polynomial of two variables; *x* and *y*.

5. $2x^5 - 3x^2 - 5$

a. $a_n x^n$ means $2 = a_5$.

b. $a_3 = 0$ (Term is $0x^3$)

c. $-3 = a_2$

d. $a_0 = -5$, the constant term.

6. $5x^3 - 4x - 17$

a. $a_3 = 5$

b. $a_1 = -4$ (Term is $-4x$)

c. $a_2 = 0$

d. $-17 = a_0$

7. $4x - x^2$

When $x = -2$,

$$\begin{aligned} 4x - x^2 &= 4(-2) - (-2)^2 \\ &= -8 - 4 \\ &= -12. \end{aligned}$$

8. $10 - 6\left(4 - x\right)^2$

When $x = -1$,

$$\begin{aligned} 10 - 6\left(4 - x\right)^2 &= 10 - 6\left(4 - (-1)\right)^2 \\ &= 10 - 150 \\ &= -140. \end{aligned}$$

9. $10xy-4(x-y)^2$

When $x=5$ and $y=-2$,

$$\begin{aligned}10xy-4(x-y)^2 &= 10(5)(-2)-4(5-(-2))^2\\ &= -100-196\\ &= -296.\end{aligned}$$

10. $3x^2-4y^2-2xy$

When $x=3$ and $y=-4$,

$$\begin{aligned}3x^2-4y^2-2xy &= 3\cdot 3^2-4(-4)^2-2\cdot 3(-4)\\ &= 27-64+24\\ &= -13.\end{aligned}$$

11. $\dfrac{2x-y}{x^2-2y}$

When $x=-5$ and $y=-3$,

$$\frac{2x-y}{x^2-2y}=\frac{2(-5)-(-3)}{(-5)^2-2(-3)}=\frac{-10+3}{25+6}=-\frac{7}{31}.$$

12. $\dfrac{16y}{1-y}$

When $y=-3$,

$$\frac{16y}{1-y}=\frac{16(-3)}{1-(-3)}=\frac{-48}{4}=-12$$

13. $1.98T-1.09(1-H)(T-58)-56.8$

$$\begin{aligned}&=1.98(74.7)-1.09(1-0.80)(74.7-58)-56.8\\ &=147.906-3.6406-56.8\ =87.4654\end{aligned}$$

14. $(100,000)\left[\dfrac{0.083(0.07)}{1-(1+0.083(0.07))^{-360}}\right]$

$$=(100,000)\left[\frac{0.00581}{0.87576}\right]\approx 663.4238$$

15. $(16pq-7p^2)+(5pq+5p^2)=21pq-2p^2$

16. $(3x^3+4x^2y^2)+(3x^2y^2-7x^3)$

$$\begin{aligned}&=(3x^3-7x^3)+(4x^2y^2+3x^2y^2)\\ &=-4x^3+7x^2y^2\end{aligned}$$

17. $(4m^2-3n^2+5)-(3m^2+4n^2+8)$

$$\begin{aligned}&=4m^2-3n^2+5-3m^2-4n^2-8\\ &=(4m^2-3m^2)-(3n^2+4n^2)+5-8\\ &=m^2-7n^2-3\end{aligned}$$

18. $(4rs-2r^2s-11rs^2)-(11rs^2-2rs+4r^2s)$

$$\begin{aligned}&=4rs-2r^2s-11rs^2-11rs^2+2rs-4r^2s\\ &=(4rs+2rs)-(2r^2s+4r^2s)-(11rs^2+11rs^2)\\ &=6rs-6r^2s-22rs^2\end{aligned}$$

19. $-[8-4(q+5)+q]=-[8-4q-20+q]$

$$\begin{aligned}&=-[-12-3q]\\ &=12+3q\end{aligned}$$

20. $x^3+[3x-(x^3-3x)]=x^3+[3x-x^3+3x]$

$$\begin{aligned}&=x^3+3x-x^3+3x\\ &=6x\end{aligned}$$

21. $x^2-[x-(x^2-1)+1-(1-x^2)]+x$

$$\begin{aligned}&=x^2-[x-x^2+1+1-1+x^2]+x\\ &=x^2-(x+1)+x\\ &=x^2-x-1+x\\ &=x^2-1\end{aligned}$$

22. $y^3-[y^2-(y^3+y^2)]-[y^3+(1-y^2)]$

$$\begin{aligned}&=y^3-[y^2-y^3-y^2]-[y^3+1-y^2]\\ &=y^3-y^2+y^3+y^2-y^3-1+y^2\\ &=y^3+y^2-1\end{aligned}$$

23. $(5x^3)(7x^2)=35x^{3+2}=35x^5$

24. $(-3x^2y)(2xy^3)(4x^2y^2)$

$$\begin{aligned}&=(-3)\cdot(2)\cdot(4)\cdot x^2\cdot x\cdot x^2\cdot y\cdot y^3\cdot y^2\\ &=-24x^5y^6\end{aligned}$$

25. $(39r^3s^2)\div(13r^2s)=3r^{3-2}s^{2-1}=3rs$

26. $(-15m^3n)\div(5mn^4)=\dfrac{-15m^3n}{5mn^4}=\dfrac{-3m^2}{n^3}$

27. $ax^2(2x^2+ax+ab)=2ax^4+a^2x^3+a^2bx^2$

28. $-3\left(3-x^2\right)=-9+3x^2=3x^2-9$

29. $(3y+4)(2y-3)=6y^2-9y+8y-12$
$=6y^2-y-12$

30. $(4x-1)(x-3)$
$=4x(x)+4x(-3)+(-1)(x)+(-1)(-3)$
$=4x^2-12x-x+3$
$=4x^2-13x+3$

31. $6\left(1-2x^2\right)\left(2-x^2\right)$
$=6\left(2-x^2-4x^2+2x^4\right)$
$=6\left(2-5x^2+2x^4\right)$
$=12x^4-30x^2+12$

32. $2\left(x^3+3\right)\left(2x^3-5\right)=2\left(2x^6-5x^3+6x^3-15\right)$
$=2\left(2x^6+x^3-15\right)$
$=4x^6+2x^3-30$

33. $\left(4x+3\right)^2=16x^2+2\left(4x\right)\left(3\right)+9=16x^2+24x+9$

34. $(2y+5)^2=(2y)^2+2(2y)(5)+(5)^2$
$=4y^2+20y+25$

35. $(0.1-4x)(0.1+4x)=(0.1)^2-(4x)^2$
$=0.01-16x^2$

36. $\left(x^3y^3-0.3\right)^2=\left(x^3y^3\right)^2+2\left(x^3y^3\right)(-0.3)+(-0.3)^2$
$=x^6y^6-0.6x^3y^3+0.09$

37. $9(2x+1)(2x-1)=9\left[(2x)^2-1^2\right]=9\left[4x^2-1\right]$
$=36x^2-9$

38. $3(5y+2)(5y-2)=3\left[(5y)^2-2^2\right]=3\left[25y^2-4\right]$
$=75y^2-12$

39. $\left(x^2-\frac{1}{2}\right)^2=x^4+2(x^2)\left(-\frac{1}{2}\right)+\left(-\frac{1}{2}\right)^2$
$=x^4-x^2+\frac{1}{4}$

40. $\left(\frac{2}{3}+x\right)\left(\frac{2}{3}-x\right)=\left(\frac{2}{3}\right)^2-(x)^2=\frac{4}{9}-x^2$

41. $(0.1x-2)(x+0.05)=0.1x^2+0.005x-2x-0.10$
$=0.1x^2-1.995x-0.10$

42. $(6.2x+4.1)(6.2x-4.1)=(6.2x)^2-(4.1)^2$
$=38.44x^2-16.81$

43.
$$\begin{array}{r} x^2+2x+4 \\ \underline{\qquad\quad x-2} \\ -2x^2-4x-8 \\ \underline{x^3+2x^2+4x\qquad\quad} \\ x^3\qquad\quad -8 \end{array}$$

44.
$$\begin{array}{r} a^2-ab+b^2 \\ \underline{\quad a+b\quad} \\ a^3-a^2b+ab^2 \\ \underline{\qquad a^2b-ab^2+b^3} \\ a^3\qquad\quad +b^3 \end{array}$$

45.
$$\begin{array}{r} x^5-2x^3+5 \\ \underline{\qquad\qquad\quad x^3+5x} \\ 5x^6-10x^4\qquad +25x \\ \underline{x^8-2x^6\qquad +5x^3\qquad\quad} \\ x^8+3x^6-10x^4+5x^3+25x \end{array}$$

46.
$$\begin{array}{r} x^7-2x^4-5x^2+5 \\ \underline{\quad x^3-1\quad} \\ x^{10}-2x^7-5x^5\qquad +5x^3 \\ \underline{\quad -x^7\qquad +2x^4\qquad +5x^2-5} \\ x^{10}-3x^7-5x^5+2x^4+5x^3+5x^2-5 \end{array}$$

47. $\frac{18m^2n+6m^3n+12m^4n^2}{6m^2n}$
$=\frac{18m^2n}{6m^2n}+\frac{6m^3n}{6m^2n}+\frac{12m^4n^2}{6m^2n}=3+m+2m^2n$

48. $\frac{16x^2+4xy^2+8x}{4xy}=\frac{16x^2}{4xy}+\frac{4xy^2}{4xy}+\frac{8x}{4xy}$
$=\frac{4x}{y}+y+\frac{2}{y}$

49. $\dfrac{24x^8y^4+15x^5y-6x^7y}{9x^5y^2}$

$=\dfrac{24x^8y^4}{9x^5y^2}+\dfrac{15x^5y}{9x^5y^2}-\dfrac{6x^7y}{9x^5y^2}=\dfrac{8x^3y^2}{3}+\dfrac{5}{3y}-\dfrac{2x^2}{3y}$

50. $\dfrac{27x^2y^2-18xy+9xy^2}{6xy}$

$=\dfrac{27x^2y^2}{6xy}-\dfrac{18xy}{6xy}+\dfrac{9xy^2}{6xy}=\dfrac{9xy}{2}-3+\dfrac{3y}{2}$

51. $(x+1)^3=x^3+3(x^2)(1)+3(x)(1)^2+1^3$

$=x^3+3x^2+3x+1$

52. $(x-3)^3=x^3-3(3)(x^2)+3(3)^2x-3^3$

$=x^3-9x^2+27x-27$

53. $(2x-3)^3=(2x)^3-3(2x)^2(3)+3(2x)(3)^2-3^3$

$=8x^3-36x^2+54x-27$

54. $(3x+4)^3=(3x)^3+3(4)(3x)^2+3(4)^2(3x)+(4)^3$

$=27x^3+108x^2+144x+64$

55.
$$\begin{array}{r} x^2-2x+5 \\ x+2\overline{)\,x^3\qquad +x-1} \\ \underline{x^3+2x^2} \\ -2x^2+\;x-1 \\ \underline{-2x^2-4x} \\ 5x-\;1 \\ \underline{5x+10} \\ -11 \end{array}$$

Quotient: $x^2-2x+5-\dfrac{11}{x+2}$

56.
$$\begin{array}{r} x^4-x^3+x^2-x+6 \\ x+1\overline{)\,x^5+0x^4+0x^3+0x^2+5x-7} \\ \underline{x^5+\;x^4} \\ -x^4+0x^3 \\ \underline{-x^4-\;x^3} \\ x^3+0x^2 \\ \underline{x^3+\;x^2} \\ -x^2+5x \\ \underline{-x^2-\;x} \\ 6x-7 \\ \underline{6x+6} \\ -13 \end{array}$$

Quotient: $x^4-x^3+x^2-x+6-\dfrac{13}{x+1}$

57.
$$\begin{array}{r} x^2+3x-1 \\ x^2+1\overline{)\,x^4+3x^3\qquad -x+1} \\ \underline{x^4\qquad +x^2} \\ 3x^3-x^2-x+1 \\ \underline{3x^3\qquad +3x} \\ -x^2-4x+1 \\ \underline{-x^2\qquad -1} \\ -4x+2 \end{array}$$

Quotient: $x^2+3x-1+\dfrac{-4x+2}{x^2+1}$

58.
$$\begin{array}{r} x\;+5 \\ x^2-2\overline{)\,x^3+5x^2+0x-6} \\ \underline{x^3\qquad -2x} \\ 5x^2+2x-6 \\ \underline{5x^2\qquad -10} \\ 2x+4 \end{array}$$

Quotient: $x+5+\dfrac{2x+4}{x^2-2}$

59. a. $(3x-2)^2-3x-2(3x-2)+5$

$=9x^2-12x+4-3x-6x+4+5$

$=9x^2-21x+13$

b. $(3x-2)^2-(3x-2)(3x-2)+5$

$=(3x-2)^2-(3x-2)^2+5$

$=5$

60. a. $(2x-3)(3x+2)-(5x-2)(x-3)$

$=6x^2-5x-6-(5x^2-17x+6)$

$=6x^2-5x-6-5x^2+17x-6$

$=x^2+12x-12$

b. $2x-3(3x+2)-5x-2(x-3)$

$=2x-9x-6-5x-2x+6$

$=-14x$

61. $x^{1/2}\left(x^{1/2}+2x^{3/2}\right)=x^{2/2}+2x^{4/2}=x+2x^2$

62. $x^{-2/3}\left(x^{5/3}-x^{-1/3}\right)=\left(x^{-2/3}\right)\left(x^{5/3}\right)+\left(x^{-2/3}\right)\left(-x^{-1/3}\right)$
$=x^{3/3}-x^{-3/3}$
$=x-\dfrac{1}{x}$

63. $\left(x^{1/2}+1\right)\left(x^{1/2}-2\right)=x-2x^{1/2}+x^{1/2}-2$
$=x-x^{1/2}-2$

64. $\left(x^{1/3}-x^{1/2}\right)\left(4x^{2/3}-3x^{3/2}\right)=\left(x^{1/3}\right)\left(4x^{2/3}\right)+\left(x^{1/3}\right)\left(-3x^{3/2}\right)+\left(-x^{1/2}\right)\left(4x^{2/3}\right)+\left(-x^{1/2}\right)\left(-3x^{3/2}\right)$
$=4x^{3/3}-3x^{11/6}-4x^{7/6}+3x^{4/2}$
$=4x-3x^{11/6}-4x^{7/6}+3x^{2}$

65. $\left(\sqrt{x}+3\right)\left(\sqrt{x}-3\right)=\left(\sqrt{x}\right)^{2}-(3)^{2}=x-9$

66. $\left(x^{1/5}+x^{1/2}\right)\left(x^{1/5}-x^{1/2}\right)=\left(x^{1/5}\right)^{2}-\left(x^{1/2}\right)^{2}=x^{2/5}-x$

67. $(2x+1)^{1/2}\left[(2x+1)^{3/2}-(2x+1)^{-1/2}\right]=(2x+1)^{2}-(2x+1)^{0}=4x^{2}+4x+1-1=4x^{2}+4x$

68. $(4x-3)^{-5/3}\left[(4x-3)^{8/3}+3(4x-3)^{5/3}\right]=(4x-3)^{-5/3}(4x-3)^{8/3}+(4x-3)^{-5/3}(3)(4x-3)^{5/3}$
$=(4x-3)^{3/3}+3(4x-3)^{0}=4x-3+3=4x$

69. $R=55x$

70. $R=215x,\ C=65x+15{,}000$

a. Profit $=P=215x-(65x+15{,}000)$
$=150x-15{,}000$

b. x = 1000: $P=150(1000)-15{,}000$
$=150{,}000-15{,}000$
$=\$135{,}000$

71. **a.** $C=49.95+0.49x$

b. $C=49.95+0.49(132)=\$114.63$

72. **a.** $C=1500+18.50x$

b. $R=45.50x$

c. $P=45.50x-(1500+18.50x)$
$=27x-1500$

73. **a.** $4000-x$

b. $0.10x$

c. $0.08(4000-x)$

d. $0.10x+0.08(4000-x)$ or $320+0.02x$

74. **a.** y = 10 cc – amount of 20% solution = $10-x$

b. Amount of ingredient = (% concentration) · (#cc) = $0.20x$

c. Amount of ingredient in y = (% concentration) (#cc) = $0.05(10-x)$

d. Total amount of ingredient in mixture is (b) + (c).
Total amount:
$0.20x+0.05(10-x)=0.50+0.15x$

75. $V=x(15-2x)(10-2x)$

Exercises 0.6

1. $9ab-12a^{2}b+18b^{2}=3b\left(3a-4a^{2}+6b\right)$

2. $8a^{2}b-160x+4bx^{2}=4\left(2a^{2}b-40x+bx^{2}\right)$

3. $4x^{2}+8xy^{2}+2xy^{3}=2x\left(2x+4y^{2}+y^{3}\right)$

4. $12y^{3}z+4yz^{2}-8y^{2}z^{3}=4yz\left(3y^{2}+z-2yz^{2}\right)$

5. $\left(7x^{3}-14x^{2}\right)+(2x-4)=7x^{2}(x-2)+2(x-2)$
$=(x-2)\left(7x^{2}+2\right)$

6. $5y-20-x^2y+4x^2=(5y-20)+(-x^2y+4x^2)$
$=5(y-4)-x^2(y-4)$
$=(5-x^2)(y-4)$

7. $6x-6m+xy-my=(6x-6m)+(xy-my)$
$=6(x-m)+y(x-m)$
$=(x-m)(6+y)$

8. $x^3-x^2-5x+5=x^2(x-1)-5(x-1)$
$=(x-1)(x^2-5)$

9. $x^2+8x+12=(x+6)(x+2)$

10. $x^2-2x-8=(x-4)(x+2)$

11. $x^2-15x-16=(x-16)(x+1)$

12. $x^2-21x+20=(x-20)(x-1)$

13. $7x^2-10x-8$
$7x^2\cdot 8=56x^2$
The factors $-14x$ and $+4x$ give a sum of $-10x$.
$7x^2-10x-8=7x^2-14x+4x-8$
$=7x(x-2)+4(x-2)$
$=(x-2)(7x+4)$

14. $12x^2+11x+2$
Two expressions whose product is $(12x^2)(2)=24x^2$ and whose sum is $11x$ are $8x$ and $3x$.
So, $12x^2+11x+2=12x^2+3x+8x+2$
$=3x(4x+1)+2(4x+1)$
$=(4x+1)(3x+2).$

15. $x^2-10x+25=x^2-2\cdot 5x+5^2=(x-5)^2$

16. $4y^2+12y+9$
Two expressions whose product is $(4y^2)(9)=36y^2$ and whose sum is 12y are $6y$ and $6y$.
So, $4y^2+12y+9=4y^2+6y+6y+9$
$=2y(2y+3)+3(2y+3)$
$=(2y+3)(2y+3)$
$=(2y+3)^2.$

17. $49a^2-144b^2=(7a)^2-(12b)^2$
$=(7a+12b)(7a-12b)$

18. $16x^2-25y^2=(4x)^2-(5y)^2$
$=(4x-5y)(4x+5y)$

19. a. $9x^2+21x-8$
$9x^2(-8)=-72x^2$
The factors $24x$ and $-3x$ give a sum of $21x$.
$9x^2+21x-8=9x^2+24x-3x-8$
$=3x(3x+8)-1(3x+8)$
$=(3x+8)(3x-1)$

b. $9x^2+22x+8$
$9x^2\cdot 8=72x^2$
The factors $18x$ and $4x$ give a sum of $22x$.
$9x^2+22x+8=9x^2+18x+4x+8$
$=9x(x+2)+4(x+2)$
$=(x+2)(9x+4)$

20. a. $10x^2-99x-63$
$10x^2\cdot(-63)=-630x^2$
The factors $-105x$ and $6x$ give a sum of $-99x$.
$10x^2-99x-63=10x^2-105x+6x-63$
$=5x(2x-21)+3(2x-21)$
$=(2x-21)(5x+3)$

b. $10x^2-27x-63$
Two expressions whose product is $(10x^2)(-63)=-630x^2$ and whose sum is $-27x$ are $-42x$ and $15x$. So,
$10x^2-27x-63=10x^2+15x-42x-63$
$=5x(2x+3)-21(2x+3)$
$=(2x+3)(5x-21).$

c. $10x^2+61x-63$
$10x^2\cdot(-63)=-630x^2$
The factors $70x$ and $-9x$ give a sum of $61x$.
$10x^2+61x-63=10x^2+70x-9x-63$
$=10x(x+7)-9(x+7)$
$=(x+7)(10x-9)$

d. $10x^2+9x-63$
Two expressions whose product is $(10x^2)(-63)=-630x^2$ and whose sum is $9x$ are $30x$ and $-21x$. So,
$10x^2+9x-63=10x^2+30x-21x-63$
$=10x(x+3)-21(x+3)$
$=(x+3)(10x-21).$

21. $4x^2-x=x(4x-1)$

22. $2x^5+18x^3=2x^3(x^2+9)$

23. $x^3+4x^2-5x-20=x^2(x+4)-5(x+4)$
$=(x+4)(x^2-5)$

24. $x^3-2x^2-3x+6=(x^3-2x^2)+(-3x+6)$
$=x^2(x-2)-3(x-2)$
$=(x^2-3)(x-2)$

25. $x^2-x-6=(x-3)(x+2)$
Note that two numbers whose product is -6 and whose sum is -1 are -3 and 2.

26. $x^2+6x+8=(x+4)(x+2)$
Since two numbers whose product is 8 and whose sum is 6 are 4 and 2.

27. $2x^2-8x-42=2(x^2-4x-21)=2(x-7)(x+3)$

28. $3x^2-21x+36=3(x^2-7x+12)$
Two numbers whose product is 12 and whose sum is -7 are -3 and -4. So,
$3(x^2-7x+12)=3(x-3)(x-4).$

29. $2x^3-8x^2+8x=2x(x^2-4x+4)$
$=2x(x^2-2\cdot2x+2^2)$
$=2x(x-2)^2$

30. $x^3+16x^2+64x=x(x^2+16x+64)=x(x+8)^2$

31. $2x^2+x-6$
$2x^2\cdot(-6)=-12x^2$
The factors $4x$ and $-3x$ give a sum of x.
$2x^2+x-6=2x^2+4x-3x-6$
$=2x(x+2)-3(x+2)$
$=(2x-3)(x+2)$

32. $2x^2+13x+6$
Two expressions whose product is $(2x^2)(6)=12x^2$ and whose sum is $13x$ are $12x$ and x. So,
$2x^2+13x+6=2x^2+12x+x+6$
$=2x(x+6)+1(x+6)$
$=(x+6)(2x+1).$

33. $3x^2+3x-36=3(x^2+x-12)=3(x+4)(x-3)$

34. $4x^2-8x-60=4(x^2-2x-15)$
Two numbers whose product is -15 and whose sum is -2 are -5 and 3. So,
$4(x^2-2x-15)=4(x-5)(x+3).$

35. $2x^3-8x=2x(x^2-4)=2x(x+2)(x-2)$

36. $16z^2-81w^2=(4z)^2-(9w)^2=(4z-9w)(4z+9w)$

37. $10x^2+19x+6$
$10x^2\cdot6=60x^2$
The factors $4x$ and $15x$ give a sum of $19x$.
$10x^2+19x+6=10x^2+4x+15x+6$
$=2x(5x+2)+3(5x+2)$
$=(5x+2)(2x+3)$

38. $6x^2+67x-35$

Two expressions whose product is $(6x^2)(-35)=-210x^2$ and whose sum is $67x$ are $70x$ and $-3x$. So,

$$\begin{aligned}6x^2+67x-35 &= 6x^2-3x+70x-35\\ &= 3x(2x-1)+35(2x-1)\\ &= (2x-1)(3x+35).\end{aligned}$$

39. $9-47x+10x^2$

$9\cdot 10x^2=90x^2$

The factors $-45x$ and $-2x$ give a sum of $-47x$.

$$\begin{aligned}9-47x+10x^2 &= 9-45x-2x+10x^2\\ &= 9(1-5x)-2x(1-5x)\\ &= (1-5x)(9-2x)\\ &\text{ or } (5x-1)(2x-9)\end{aligned}$$

40. $10x^2+21x-10$

Two expressions whose product is $(10x^2)(-10)=-100x^2$ and whose sum is $21x$ are $25x$ and $-4x$. So,

$$\begin{aligned}10x^2+21x-10 &= 10x^2+25x-4x-10\\ &= 5x(2x+5)-2(2x+5)\\ &= (2x+5)(5x-2)\end{aligned}$$

41. $y^4-16x^4=\left(y^2\right)^2-\left(4x^2\right)^2$

$$\begin{aligned}&= \left(y^2-4x^2\right)\left(y^2+4x^2\right)\\ &= (y-2x)(y+2x)\left(y^2+4x^2\right)\end{aligned}$$

42. $x^8-81=\left(x^4\right)^2-9^2$

$$\begin{aligned}&= \left(x^4+9\right)\left(x^4-9\right)\\ &= \left(x^4+9\right)\left(x^2+3\right)\left(x^2-3\right)\end{aligned}$$

43. $x^4-8x^2+16=\left(x^2\right)^2-2\cdot 4x^2+4^2=\left(x^2-4\right)^2$

$$\begin{aligned}&= [(x-2)(x+2)]^2\\ &= (x-2)^2(x+2)^2\end{aligned}$$

44. $81-18x^2+x^4$

Two expressions whose product is $(81)(x^4)=81x^4$ and whose sum is $-18x^2$ are $-9x^2$ and $-9x^2$. So,

$$\begin{aligned}x^4-18x^2+81 &= x^4-9x^2-9x^2+81\\ &= x^2\left(x^2-9\right)-9\left(x^2-9\right)\\ &= \left(x^2-9\right)\left(x^2-9\right)\\ &= (x-3)(x+3)(x-3)(x+3)\\ &= (x-3)^2(x+3)^2.\end{aligned}$$

45. $4x^4-5x^2+1=\left(4x^2-1\right)\left(x^2-1\right)$

$$= (2x+1)(2x-1)(x+1)(x-1)$$

46. x^4-3x^2-4

Two expression whose product is $(x^4)(-4)=-4x^4$ and whose sum is $-3x^2$ are $-4x^2$ and x^2. So,

$$\begin{aligned}x^4-3x^2-4 &= x^4+x^2-4x^2-4\\ &= x^2\left(x^2+1\right)-4\left(x^2+1\right)\\ &= \left(x^2+1\right)(x+2)(x-2)\end{aligned}$$

47. $x^{3/2}+x^{1/2}=x^{1/2}(x^{2/2}+1)$

$$\begin{aligned}&= x^{1/2}(x+1)\\ &?= x+1\end{aligned}$$

48. $2x^{1/4}+4x^{3/4}=2x^{1/4}\left(1+2x^{2/4}\right)$

$$\begin{aligned}&= 2x^{1/4}\left(1+2x^{1/2}\right)\\ &?= 1+2x^{1/2}\end{aligned}$$

49. $x^{-3}+x^{-2}=x^{-3}\left(1+x^1\right)$

$$\begin{aligned}&= x^{-3}(1+x)\\ &?= 1+x\end{aligned}$$

50. $x^{-1}-x=x^{-1}\left(1-x^2\right)$

$$?= 1-x^2$$

51. $x^3+3x^2+3x+1=(x+1)^3$

52. The expression $x^3+6x^2+12x+8$ is a perfect cube $(a+b)^3$ with $a = x$ and $b = 2$. So,

$$x^3+6x^2+12x+8=(x+2)^3.$$

53. $x^3-12x^2+48x-64 = x^3-3\left(4x^2\right)+3(16x)-4^3$
$= x^3-3x^2(4)+3x(4)^2-4^3$
$= (x-4)^3$

54. The expression $y^3-9y^2+27y-27$ is a perfect cube $(a-b)^3$ with $a = y$ and $b = 3$. So, $y^3-9y^2+27y-27=(y-3)^3$.

55. $x^3-64 = x^3-4^3 = (x-4)\left(x^2+4x+16\right)$

56. $8x^3-1=(2x)^3-(1)^3=(2x-1)\left(4x^2+2x+1\right)$

57. $27+8x^3=3^3+(2x)^3=(3+2x)\left(9-6x+4x^2\right)$

58. $a^3+216=(a)^3+(6)^3=(a+6)\left(a^2-6a+36\right)$

59. $P+Prt=P(1+rt)$

60. $R=\dfrac{cm^2}{2}-\dfrac{m^3}{3}=m^2\left(\dfrac{c}{2}-\dfrac{m}{3}\right)$

61. $S=cm-m^2=m(c-m)$

62. $V=64x-32x^2+4x^3=4x\left(16-8x+x^2\right)$
$=4x(4-x)^2$

63. a. In the form px we have $p(10,000-100p)$.
$x = 10{,}000 - 100p$
b. If $p=38$, then $x=10,000-100\cdot 38=6200$.

64. $(R+r)^2-2r(R+r)=(R+r)(R+r-2r)$
$=(R+r)(R-r)$

65. a. $R=x(300-x)$
b. $P=300-x$

66. $r^2-(r-x)^2$
$=\left[r+(r-x)\right]\left[r-(r-x)\right]$
$=[2r-x][x]=x(2r-x)$

Exercises 0.7

1. $\dfrac{18x^3y^3}{9x^3z}=\dfrac{2x^3y^3}{x^3z}=\dfrac{2y^3}{z}$

2. $\dfrac{15a^4b^5}{30a^3b}=\dfrac{15a^3b(ab^4)}{15a^3b(2)}=\dfrac{ab^4}{2}$

3. $\dfrac{x-3y}{3x-9y}=\dfrac{1(x-3y)}{3(x-3y)}=\dfrac{1}{3}$

4. $\dfrac{x^2-6x+8}{x^2-16}=\dfrac{(x-4)(x-2)}{(x-4)(x+4)}=\dfrac{x-2}{x+4}$

5. $\dfrac{x^2-2x+1}{x^2-4x+3}=\dfrac{(x-1)(x-1)}{(x-3)(x-1)}=\dfrac{x-1}{x-3}$

6. $\dfrac{x^2-5x+6}{9-x^2}=\dfrac{(x-3)(x-2)}{(3-x)(3+x)}$
$=\dfrac{-(3-x)(x-2)}{(3-x)(3+x)}$
$=\dfrac{-(x-2)}{x+3}$

7. $\dfrac{6x^3}{8y^3}\cdot\dfrac{16x}{9y^2}\cdot\dfrac{15y^4}{x^3}=\dfrac{6}{y^3}\cdot\dfrac{2x}{9y^2}\cdot\dfrac{15y^4}{1}=\dfrac{2}{1}\cdot\dfrac{2x}{3y^2}\cdot\dfrac{15y}{1}$
$=\dfrac{2}{1}\cdot\dfrac{2x}{y}\cdot\dfrac{5}{1}$
$=\dfrac{20x}{y}$

8. $\dfrac{25ac^2}{15a^2c}\cdot\dfrac{4ad^4}{15abc^3}=\dfrac{100a^2c^2d^4}{225a^3bc^4}=\dfrac{25a^2c^2(4d^4)}{25a^2c^2(9abc^2)}$
$=\dfrac{4d^4}{9abc^2}$

9. $\dfrac{8x-16}{x-3}\cdot\dfrac{4x-12}{3x-6}=\dfrac{8(x-2)}{x-3}\cdot\dfrac{4(x-3)}{3(x-2)}=\dfrac{8\cdot 4}{3}=\dfrac{32}{3}$

10. $\dfrac{(x^2-4)}{1}\cdot\dfrac{(2x-3)}{(x+2)}=\dfrac{(x-2)(x+2)}{1}\cdot\dfrac{(2x-3)}{(x+2)}$
$=(x-2)(2x-3)$

11. $\dfrac{x^2+7x+12}{3x^2+13x+4}\cdot\dfrac{9x+3}{1}$

$=\dfrac{(x+4)(x+3)}{(3x+1)(x+4)}\cdot\dfrac{3(3x+1)}{1}$

$=3(x+3)$

$=3x+9$

12. $\dfrac{4x+4}{x-4}\cdot\dfrac{x^2-6x+8}{8x^2+8x}=\dfrac{4(x+1)}{x-4}\cdot\dfrac{(x-4)(x-2)}{8x(x+1)}$

$=\dfrac{x-2}{2x}$

13. $\dfrac{x^2-x-2}{2x^2-8}\cdot\dfrac{18-2x^2}{x^2-5x+4}\cdot\dfrac{x^2-2x-8}{x^2-6x+9}$

$=\dfrac{(x-2)(x+1)}{2(x^2-4)}\cdot\dfrac{-2(x^2-9)}{(x-4)(x-1)}\cdot\dfrac{(x-4)(x+2)}{(x-3)(x-3)}$

$=\dfrac{(x-2)(x+1)}{2(x-2)(x+2)}\cdot\dfrac{-2(x-3)(x+3)}{(x-1)}\cdot\dfrac{(x+2)}{(x-3)(x-3)}$

$=-\dfrac{(x+1)(x+3)}{(x-1)(x-3)}$

14. $\dfrac{x^2-5x-6}{x^2-5x+4}\cdot\dfrac{x^2-x-12}{x^3-6x^2}\cdot\dfrac{x-x^3}{x^2-2x+1}$

$=\dfrac{(x-6)(x+1)}{(x-4)(x-1)}\cdot\dfrac{(x-4)(x+3)}{x^2(x-6)}\cdot\dfrac{x(1-x)(1+x)}{(x-1)(x-1)}$

$=\dfrac{(x+1)^2(x+3)(1-x)}{x(x-1)^3}$

$=\dfrac{-(x+1)^2(x+3)(x-1)}{x(x-1)^3}$

$=\dfrac{-(x+1)^2(x+3)}{x(x-1)^2}$

15. $\dfrac{15ac^2}{7bd}\div\dfrac{4a}{14b^2d}=\dfrac{15ac^2}{7bd}\cdot\dfrac{14b^2d}{4a}$

$=\dfrac{15c^2}{1}\cdot\dfrac{2b}{4}$

$=\dfrac{15bc^2}{2}$

16. $\dfrac{16}{x-2}\div\dfrac{4}{3x-6}=\dfrac{16}{x-2}\cdot\dfrac{3x-6}{4}$

$=\dfrac{16}{x-2}\cdot\dfrac{3(x-2)}{4}$

$=12$

17. $\dfrac{y^2-2y+1}{7y^2-7y}\div\dfrac{y^2-4y+3}{35y^2}$

$=\dfrac{y^2-2y+1}{7y(y-1)}\cdot\dfrac{35y^2}{y^2-4y+3}$

$=\dfrac{(y-1)(y-1)}{7y(y-1)}\cdot\dfrac{35y^2}{(y-3)(y-1)}$

$=\dfrac{5y}{y-3}$

18. $\dfrac{6x^2}{4x^2y-12xy}\div\dfrac{3x^2+12x}{x^2+x-12}$

$=\dfrac{6x^2}{4xy(x-3)}\cdot\dfrac{(x+4)(x-3)}{3x(x+4)}$

$=\dfrac{2}{4y}$

$=\dfrac{1}{2y}$

19. $\dfrac{x^2-x-6}{1}\div\dfrac{9-x^2}{x^2-3x}$

$=\dfrac{x^2-x-6}{1}\cdot\dfrac{x^2-3x}{-1(x^2-9)}$

$=\dfrac{(x-3)(x+2)}{1}\cdot\dfrac{x(x-3)}{-1(x-3)(x+3)}$

$=\dfrac{-x(x-3)(x+2)}{x+3}$

20. $\dfrac{2x^2+7x+3}{4x^2-1}\div(x+3)=\dfrac{(2x+1)(x+3)}{(2x-1)(2x+1)}\cdot\dfrac{1}{x+3}$

$=\dfrac{1}{2x-1}$

21. $\dfrac{2x}{x^2-x-2}-\dfrac{x+2}{x^2-x-2}=\dfrac{2x-x-2}{(x-2)(x+1)}$

$=\dfrac{x-2}{(x-2)(x+1)}$

$=\dfrac{1}{x+1}$

22. $\dfrac{4}{9-x^2}-\dfrac{x+1}{9-x^2}=\dfrac{4-(x+1)}{9-x^2}=\dfrac{4-x-1}{9-x^2}$

$=\dfrac{3-x}{(3+x)(3-x)}$

$=\dfrac{1}{3+x}$

23. $\dfrac{a}{a-2}-\dfrac{a-2}{a}=\dfrac{a}{a-2}\cdot\dfrac{a}{a}-\dfrac{a-2}{a}\cdot\dfrac{a-2}{a-2}$

$=\dfrac{a^2-(a^2-4a+4)}{a(a-2)}$

$=\dfrac{4a-4}{a(a-2)}$

$=\dfrac{4(a-1)}{a(a-2)}$

24. $x-\dfrac{2}{x-1}=\dfrac{(x-1)x}{x-1}-\dfrac{2}{x-1}$

$=\dfrac{(x-1)(x)-2}{x-1}$

$=\dfrac{x^2-x-2}{x-1}$

$=\dfrac{(x+1)(x-2)}{x-1}$

25. $\dfrac{x}{x+1}-x+1=\dfrac{x}{x+1}-\dfrac{x}{1}\cdot\dfrac{x+1}{x+1}+\dfrac{1}{1}\cdot\dfrac{x+1}{x+1}$

$=\dfrac{x-x^2-x+x+1}{x+1}$

$=\dfrac{-x^2+x+1}{x+1}$

26. $\dfrac{x-1}{x+1}-\dfrac{2}{x^2+x}=\dfrac{x-1}{x+1}-\dfrac{2}{x(x+1)}$

$=\dfrac{x(x-1)}{x(x+1)}-\dfrac{2}{x(x+1)}$

$=\dfrac{x^2-x-2}{x(x+1)}=\dfrac{(x+1)(x-2)}{x(x+1)}$

$=\dfrac{x-2}{x}$

27. $\dfrac{4a}{3x+6}+\dfrac{5a^2}{4x+8}=\dfrac{4a}{3(x+2)}+\dfrac{5a^2}{4(x+2)}=\dfrac{4a}{3(x+2)}\cdot\dfrac{4}{4}+\dfrac{5a^2}{4(x+2)}\cdot\dfrac{3}{3}=\dfrac{16a+15a^2}{12(x+2)}$

28. $\dfrac{b-1}{b^2+2b}+\dfrac{b}{3b+6}=\dfrac{b-1}{b(b+2)}+\dfrac{b}{3(b+2)}=\dfrac{3(b-1)}{3b(b+2)}+\dfrac{b^2}{3b(b+2)}=\dfrac{b^2+3b-3}{3b(b+2)}$

29. $\dfrac{3x-1}{2x-4}+\dfrac{4x}{3x-6}-\dfrac{x-4}{5x-10}=\dfrac{3x-1}{2(x-2)}+\dfrac{4x}{3(x-2)}-\dfrac{x-4}{5(x-2)}$

$$= \frac{3x-1}{2(x-2)} \cdot \frac{3 \cdot 5}{3 \cdot 5} + \frac{4x}{3(x-2)} \cdot \frac{2 \cdot 5}{2 \cdot 5} - \frac{(x-4)}{5(x-2)} \cdot \frac{3 \cdot 2}{3 \cdot 2}$$
$$= \frac{(45x-15)+40x-6x+24}{30(x-2)}$$
$$= \frac{79x+9}{30(x-2)}$$

30. $\frac{2x+1}{4x-2} + \frac{5}{2x} - \frac{x+4}{2x^2-x} = \frac{2x+1}{2(2x-1)} + \frac{5}{2x} - \frac{x+4}{x(2x-1)}$
$$= \frac{x(2x+1)}{2x(2x-1)} + \frac{5(2x-1)}{2x(2x-1)} - \frac{2(x+4)}{2x(2x-1)}$$
$$= \frac{2x^2+x+10x-5-2x-8}{2x(2x-1)}$$
$$= \frac{2x^2+9x-13}{2x(2x-1)}$$

31. $\frac{x}{x^2-4} + \frac{4}{x^2-x-2} - \frac{x-2}{x^2+3x+2} = \frac{x}{(x+2)(x-2)} + \frac{4}{(x-2)(x+1)} - \frac{x-2}{(x+2)(x+1)}$
$$= \frac{x}{(x+2)(x-2)} \cdot \frac{x+1}{x+1} + \frac{4}{(x-2)(x+1)} \cdot \frac{x+2}{x+2} - \frac{x-2}{(x+2)(x+1)} \cdot \frac{x-2}{x-2}$$
$$= \frac{(x^2+x)+(4x+8)(x^2-4x+4)}{(x+2)(x+1)(x-2)} = \frac{9x+4}{(x+2)(x+1)(x-2)}$$

32. $\frac{3x^2}{x^2-4} + \frac{2}{x^2-4x+4} - 3 = \frac{3x^2}{(x+2)(x-2)} + \frac{2}{(x-2)^2} - 3$
$$= \frac{3x^2(x-2)}{(x-2)^2(x+2)} + \frac{2(x+2)}{(x-2)^2(x+2)} - \frac{3(x-2)^2(x+2)}{(x-2)^2(x+2)}$$
$$= \frac{3x^3-6x^2+2x+4-3(x^2-4x+4)(x+2)}{(x-2)^2(x+2)}$$
$$= \frac{3x^3-6x^2+2x+4-3(x^3+2x^2-4x^2-8x+4x+8)}{(x-2)^2(x+2)}$$
$$= \frac{3x^3-6x^2+2x+4-3x^3-6x^2+12x^2+24x-12x-24}{(x-2)^2(x+2)}$$
$$= \frac{14x-20}{(x-2)^2(x+2)}$$

33. $\dfrac{-x^3+x}{\sqrt{3-x^2}}+\dfrac{2x\sqrt{3-x^2}}{1}$

$=\dfrac{-x^3+x}{\sqrt{3-x^2}}+\dfrac{2x\sqrt{3-x^2}}{1}\cdot\dfrac{\sqrt{3-x^2}}{\sqrt{3-x^2}}$

$=\dfrac{-x^3+x+2x\left(3-x^2\right)}{\sqrt{3-x^2}}$

$=\dfrac{-x^3+x+6x-2x^3}{\sqrt{3-x^2}}$

$=\dfrac{7x-3x^3}{\sqrt{3-x^2}}$

34. $\dfrac{3x^2\left(x+1\right)}{\sqrt{x^3+1}}+\sqrt{x^3+1}$

$=\dfrac{3x^2\left(x+1\right)}{\sqrt{x^3+1}}+\dfrac{\left(\sqrt{x^3+1}\right)\left(\sqrt{x^3+1}\right)}{\sqrt{x^3+1}}$

$=\dfrac{3x^2\left(x+1\right)+x^3+1}{\sqrt{x^3+1}}$

$=\dfrac{3x^3+3x^2+x^3+1}{\sqrt{x^3+1}}$

$=\dfrac{4x^3+3x^2+1}{\sqrt{x^3+1}}$

35. $\dfrac{\frac{3}{1}-\frac{2}{3}}{\frac{14}{1}}\cdot\dfrac{3}{3}=\dfrac{9-2}{14(3)}=\dfrac{7}{14(3)}=\dfrac{1}{6}$

36. $\dfrac{4}{\frac{1}{4}+\frac{1}{4}}=\dfrac{4}{\frac{1}{2}}=8$

37. $\dfrac{x+y}{\frac{1}{x}+\frac{1}{y}}=\dfrac{(x+y)}{\frac{1}{x}+\frac{1}{y}}\cdot\dfrac{xy}{xy}=\dfrac{xy(x+y)}{y+x}=xy$

38. $\dfrac{\frac{5}{2y}+\frac{3}{y}}{\frac{1}{4}+\frac{1}{3y}}\cdot\dfrac{12y}{12y}=\dfrac{30+36}{3y+4}=\dfrac{66}{3y+4}$

39. $\dfrac{2-\frac{1}{x}}{2x-\frac{3x}{x+1}}=\dfrac{\frac{2}{1}-\frac{1}{x}}{\frac{2x}{1}-\frac{3x}{x+1}}\cdot\dfrac{x(x+1)}{x(x+1)}$

$=\dfrac{2x(x+1)-1(x+1)}{2x^2(x+1)-3x(x)}$

$=\dfrac{2x^2+x-1}{2x^3-x^2}$

$=\dfrac{(2x-1)(x+1)}{x^2(2x-1)}=\dfrac{x+1}{x^2}$

40. $\dfrac{1-\frac{2}{x-2}}{x-6+\frac{10}{x+1}}\cdot\dfrac{(x-2)(x+1)}{(x-2)(x+1)}$

$=\dfrac{(x-2)(x+1)-2(x+1)}{(x-6)(x-2)(x+1)+10(x-2)}$

$=\dfrac{x^2+x-2x-2-2x-2}{(x^2-2x-6x+12)(x+1)+10(x-2)}$

$=\dfrac{x^2-3x-4}{(x^2-8x+12)(x+1)+10x-20}$

$=\dfrac{x^2-3x-4}{x^3-8x^2+12x+x^2-8x+12+10x-20}$

$=\dfrac{(x-4)(x+1)}{x^3-7x^2+14x-8}$

$=\dfrac{(x-4)(x+1)}{(x-4)(x^2-3x+2)}$

$=\dfrac{x+1}{x^2-3x+2}=\dfrac{x+1}{(x-1)(x-2)}$

41. $\dfrac{\sqrt{a}-\frac{b}{\sqrt{a}}}{a-b}=\dfrac{\frac{\sqrt{a}}{1}-\frac{b}{\sqrt{a}}}{\frac{a-b}{1}}\cdot\dfrac{\sqrt{a}}{\sqrt{a}}=\dfrac{a-b}{\sqrt{a}(a-b)}$

$=\dfrac{1}{\sqrt{a}}$ or $\dfrac{\sqrt{a}}{a}$

42. $\dfrac{\sqrt{x-1}+\frac{1}{\sqrt{x-1}}}{x}\cdot\dfrac{\sqrt{x-1}}{\sqrt{x-1}}=\dfrac{(\sqrt{x-1})^2+1}{x\sqrt{x-1}}$

$=\dfrac{x-1+1}{x\sqrt{x-1}}=\dfrac{1}{\sqrt{x-1}}$

43. a. $(2^{-2}-3^{-1})^{-1}=\left(\dfrac{1}{2^2}-\dfrac{1}{3}\right)^{-1}=\left(-\dfrac{1}{12}\right)^{-1}=-12$

b. $(2^{-1}+3^{-1})^2=\left(\dfrac{1}{2}+\dfrac{1}{3}\right)^2=\left(\dfrac{5}{6}\right)^2=\dfrac{25}{36}$

Hint: Work inside ( ) first when adding or subtracting is involved.

44. **a.** $(3^2+4^2)^{-1/2} = (9+16)^{-1/2} = 25^{-1/2} = \frac{1}{5}$

b. $(2^2+3^2)^{-1} = \frac{1}{2^2+3^2} = \frac{1}{4+9} = \frac{1}{13}$

45. $\frac{2a^{-1}-b^{-2}}{(ab^2)^{-1}} = \frac{\frac{2}{a}-\frac{1}{b^2}}{\frac{1}{ab^2}} \cdot \frac{ab^2}{ab^2} = \frac{2b^2-a}{1}$ or $2b^2-a$

46. $\frac{x^{-2}+xy^{-2}}{(x^2y)^{-2}} = \frac{\frac{1}{x^2}+\frac{x}{y^2}}{\frac{1}{x^4y^2}} \cdot \frac{x^4y^2}{x^4y^2} = \frac{x^2y^2+x^5}{1}$

$= x^2y^2 + x^5 = x^2(y^2+x^3)$

47. $\frac{1-\sqrt{x}}{1+\sqrt{x}} = \frac{1-\sqrt{x}}{1+\sqrt{x}} \cdot \frac{1-\sqrt{x}}{1-\sqrt{x}} = \frac{1-2\sqrt{x}+x}{1-x}$

48. $\frac{x-3}{x-\sqrt{3}} \cdot \frac{x+\sqrt{3}}{x+\sqrt{3}} = \frac{x^2+\sqrt{3}x-3x-3\sqrt{3}}{x^2-3}$

$= \frac{x^2+(\sqrt{3}-3)x-3\sqrt{3}}{x^2-3}$

49. $\frac{\sqrt{x+h}-\sqrt{x}}{h} = \frac{\sqrt{x+h}-\sqrt{x}}{h} \cdot \frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$

$= \frac{(x+h)-(x)}{h\left(\sqrt{x+h}+\sqrt{x}\right)} = \frac{h}{h\left(\sqrt{x+h}+\sqrt{x}\right)}$

$= \frac{1}{\sqrt{x+h}+\sqrt{x}}$

50. $\frac{\sqrt{9+2h}-3}{h} \cdot \frac{\sqrt{9+2h}+3}{\sqrt{9+2h}+3} = \frac{9+2h-9}{h(\sqrt{9+2h}+3)}$

$= \frac{2h}{h(\sqrt{9+2h}+3)} = \frac{2}{\sqrt{9+2h}+3}$

51. $\frac{1}{a}+\frac{1}{b}+\frac{1}{c} = \frac{1}{a}\cdot\frac{bc}{bc}+\frac{1}{b}\cdot\frac{ac}{ac}+\frac{1}{c}\cdot\frac{ab}{ab} = \frac{bc+ac+ab}{abc}$

52. **a.** $\frac{1}{p}+\frac{1}{q}-\frac{d}{pq} = \frac{q}{pq}+\frac{p}{pq}-\frac{d}{pq} = \frac{q+p-d}{pq}$

b. The reciprocal is $\frac{pq}{q+p-d}$.

53. **a.** Avg. cost $= \frac{4000}{x}+\frac{55}{1}+\frac{0.1x}{1}$

$= \frac{4000+55x+0.1x^2}{x}$

b. Total cost = (Avg. cost)(number of units)

$= 4000+55x+0.1x^2$

54. **a.** $\frac{40,500}{x}+190+0.2x$

$= \frac{0.2x^2+190x+40,500}{x}$

b. Total cost = (Avg. cost)(number of units)

$= 0.2x^2+190x+40,500$

55. $SV = 1+\frac{3}{t+3}-\frac{18}{(t+3)^2} = \frac{(t+3)^2+3(t+3)-18}{(t+3)^2} = \frac{t^2+6t+9+3t+9-18}{(t+3)^2} = \frac{t^2+9t}{(t+3)^2}$

56. $\frac{(1+i)^{n+1}-1}{i}-1 = \frac{(1+i)^{n+1}-1}{i}-\frac{i}{i} = \frac{(1+i)^{n+1}-1-i}{i}$

$= \frac{(1+i)^{n+1}-(1+i)^1}{i} = \frac{(1+i)\left[(1+i)^n-1\right]}{i}$

Chapter 0: Algebraic Concepts

Chapter 0 Review Exercises

1. Yes. $B = \{1, 2, 3, 4, 5, 6, 7, 8\}$. Since every element of A is also an element of B, A is a subset of B.

2. No. $3 \notin \{x : x > 3\}$

3. No. A and B are not disjoint since each set contains the element 1.

4. $A = \{1, 2, 3, 9\}$ $B' = \{2, 4, 9\}$.
$A \cup B' = \{1, 2, 3, 4, 9\}$

5. $\{4, 5, 6, 7, 8, 10\} \cap \{1, 3, 5, 6, 7, 8, 10\}$
$= \{5, 6, 7, 8, 10\}$

6. $A = \{1, 2, 3, 9\}$ $B = \{1, 3, 5, 6, 7, 8, 10\}$
$A' = \{4, 5, 6, 7, 8, 10\}$
$A' \cap B = \{5, 6, 7, 8, 10\}$
$(A' \cap B)' = \{1, 2, 3, 4, 9\}$

7. $\{4, 5, 6, 7, 8, 10\} \cup \{2, 4, 9\}$
$= \{2, 4, 5, 6, 7, 8, 9, 10\}$
$(A' \cup B')' = \{2, 4, 5, 6, 7, 8, 9, 10\}' = \{1, 3\}$
$A \cap B = \{1, 2, 3, 9\} \cap \{1, 3, 5, 6, 7, 8, 10\}$
$= \{1, 3\}$ Yes.

8. **a.** $6 + \frac{1}{3} = \frac{1}{3} + 6$ illustrates the Commutative Property of Addition.

b. $2(3 \cdot 4) = (2 \cdot 3)4$ illustrates the Associative Property of Multiplication.

c. $\frac{1}{3}(6 + 9) = 2 + 3$ illustrates the Distributive Law.

9. **a.** irrational
b. rational, integer
c. undefined

10. **a.** $\pi > 3.14$
b. $-100 < 0.1$
c. $-3 > -12$

11. $|5 - 11| = |-6| = -(-6) = 6$

12. $44 \div 2 \cdot 11 - 10^2 = 22 \cdot 11 - 100 = 242 - 100 = 142$

13. $(-3)^2 - (-1)^3 = 9 - (-1) = 10$

14. $\frac{(3)(2)(15) - (5)(8)}{(4)(10)} = \frac{90 - 40}{40} = \frac{50}{40} = \frac{5}{4}$

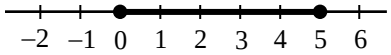
15. $2 - [3 - (2 - |-3|)] + 11 = 2 - [3 - (2 - 3)] + 11$
$= 2 - [3 - (-1)] + 11$
$= 2 - [3 + 1] + 11$
$= 2 - 4 + 11$
$= 9$

16. $-4^2 - (-4)^2 + 3 = -16 - 16 + 3 = -32 + 3 = -29$

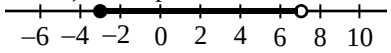
17. $\frac{4 + 3^2}{4} = \frac{4 + 9}{4} = \frac{13}{4}$

18. $\frac{(-2.91)^5}{\sqrt{3.29^5}} \approx \frac{-208.6724}{19.6331} \approx -10.62857888$

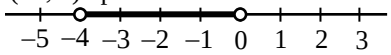
19. **a.** [0, 5], closed



b. [–3, 7), half open



c. (–4, 0) open



20. **a.** $(-1,16)$
$-1 < x < 16$
b. $[-12,8]$
$-12 \le x \le 8$
c. $x < -1$

21. **a.** $\left(\frac{3}{8}\right)^0 = 1$

b. $2^3 \cdot 2^{-5} = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$

c. $\frac{4^9}{4^3} = 4^6 = 4096$

d. $\left(\frac{1}{7}\right)^3 \left(\frac{1}{7}\right)^{-4} = \left(\frac{1}{7}\right)^{-1} = 7$

22. a. $x^5 \cdot x^{-7} = x^{5+(-7)} = x^{-2} = \dfrac{1}{x^2}$

b. $\dfrac{x^8}{x^{-2}} = x^{8-(-2)} = x^{10}$

c. $(x^3)^3 = x^{3\cdot 3} = x^9$

d. $(y^4)^{-2} = y^{(4)(-2)} = y^{-8} = \dfrac{1}{y^8}$

e. $(-y^{-3})^{-2} = y^{(-3)(-2)} = y^6$

There are other correct methods of working problems 23–28.

23. $\dfrac{-(2xy^2)^{-2}}{(3x^{-2}y^{-3})^2} = \dfrac{(-1)(2)^{-2}x^{-2}y^{-4}}{3^2x^{-4}y^{-6}}$

$= \dfrac{(-1)x^4y^6}{2^2 \cdot 3^2x^2y^4}$

$= -\dfrac{x^2y^2}{36}$

24. $\left(\dfrac{2}{3}x^2y^{-4}\right)^{-2} = \left(\dfrac{2}{3}\right)^{-2}\left(x^2\right)^{-2}\left(y^{-4}\right)^{-2}$

$= \left(\dfrac{3}{2}\right)^2\left(x^{-4}\right)\left(y^8\right)$

$= \left(\dfrac{9}{4}\right)\left(\dfrac{1}{x^4}\right)\left(y^8\right)$

$= \dfrac{9y^8}{4x^4}$

25. $\left(\dfrac{x^{-2}}{2y^{-1}}\right)^2 = \left(\dfrac{y}{2x^2}\right)^2 = \dfrac{y^2}{4x^4}$

26. $\dfrac{\left(-x^4y^{-2}z^2\right)^0}{-\left(x^4y^{-2}z^2\right)^{-2}} = \dfrac{1}{-\left(x^4\right)^{-2}\left(y^{-2}\right)^{-2}\left(z^2\right)^{-2}}$

$= \dfrac{1}{-x^{-8}y^4z^{-4}} = \dfrac{-x^8z^4}{y^4}$

27. $\left(\dfrac{x^{-3}y^4z^{-2}}{3x^{-2}y^{-3}z^{-3}}\right)^{-1} = \left(\dfrac{y^{4-(-3)}z^{-2-(-3)}}{3x^{-2-(-3)}}\right)^{-1}$

$= \left(\dfrac{y^7z}{3x}\right)^{-1} = \dfrac{3x}{y^7z}$

28. $\left(\dfrac{x}{2y}\right)\left(\dfrac{y}{x^2}\right)^{-2} = \left(\dfrac{x}{2y}\right)\left(\dfrac{x^2}{y}\right)^2$

$= \left(\dfrac{x}{2y}\right)\left(\dfrac{(x^2)^2}{y^2}\right) = \left(\dfrac{x}{2y}\right)\left(\dfrac{x^4}{y^2}\right) = \dfrac{x^5}{2y^3}$

29. a. $-\sqrt[3]{-64} = -\sqrt[3]{(-4)^3} = -(-4) = 4$

b. $\sqrt{\dfrac{4}{49}} = \sqrt{\dfrac{2^2}{7^2}} = \dfrac{2}{7}$

c. $\sqrt[7]{1.9487171} = 1.1$

30. a. $\sqrt{x} = x^{1/2}$

b. $\sqrt[3]{x^2} = x^{2/3}$

c. $1/\sqrt[4]{x} = \dfrac{1}{x^{1/4}} = x^{-1/4}$

31. a. $x^{3/7} = \sqrt[7]{x^3}$

b. $x^{-1/2} = \dfrac{1}{\sqrt{x}} = \dfrac{\sqrt{x}}{x}$

c. $-x^{3/2} = -x\sqrt{x}$

32. a. $\dfrac{5xy}{\sqrt{2x}} \cdot \dfrac{\sqrt{2x}}{\sqrt{2x}} = \dfrac{5xy\sqrt{2x}}{2x} = \dfrac{5y\sqrt{2x}}{2}$

b. $\dfrac{y}{x\sqrt[3]{xy^2}} \cdot \dfrac{\sqrt[3]{x^2y}}{\sqrt[3]{x^2y}} = \dfrac{y\sqrt[3]{x^2y}}{x\sqrt[3]{x^3y^3}}$

$= \dfrac{y\sqrt[3]{x^2y}}{x(xy)}$

$= \dfrac{y\sqrt[3]{x^2y}}{x^2y}$

$= \dfrac{\sqrt[3]{x^2y}}{x^2}$

33. $x^{1/2} \cdot x^{1/3} = x^{(3/6)+(2/6)} = x^{5/6}$

34. $\dfrac{y^{-3/4}}{y^{-7/4}} = y^{-3/4-(-7/4)} = y^{4/4} = y$

35. $x^4 \cdot x^{1/4} = x^{(16/4)+(1/4)} = x^{17/4}$

36. $\dfrac{1}{x^{-4/3} \cdot x^{-7/3}} = \dfrac{1}{x^{-11/3}} = x^{11/3}$

37. $(x^{4/5})^{1/2} = x^{(4/5)(1/2)} = x^{2/5}$

38. $(x^{1/2}y^2)^4 = (x^{1/2})^4(y^2)^4 = x^2y^8$

39. $\sqrt{12x^3y^5} = \sqrt{4x^2y^4 \cdot 3xy} = 2xy^2\sqrt{3xy}$

40. $\sqrt{1250x^6y^9} = \sqrt{625x^6y^8 \cdot 2y} = 25x^3y^4\sqrt{2y}$

41. $\sqrt[3]{24x^4y^4} \cdot \sqrt[3]{45x^4y^{10}} = \sqrt[3]{8x^3y^3 \cdot 3xy} \cdot \sqrt[3]{9x^3y^9 \cdot 5xy}$
$= 2xy\sqrt[3]{3xy} \cdot xy^3\sqrt[3]{9 \cdot 5xy}$
$= 2x^2y^4\sqrt[3]{27 \cdot 5x^2y^2}$
$= 6x^2y^4\sqrt[3]{5x^2y^2}$

42. $\sqrt{16a^2b^3} \cdot \sqrt{8a^3b^5} = \sqrt{128a^5b^8}$
$= \sqrt{64a^4b^8 \cdot 2a}$
$= 8a^2b^4\sqrt{2a}$

43. $\dfrac{\sqrt{52x^3y^6}}{\sqrt{13xy^4}} = \sqrt{4x^2y^2} = 2xy$

44. $\dfrac{\sqrt{32x^4y^3}}{\sqrt{6xy^{10}}} = \sqrt{\dfrac{16x^3}{3y^7}} = \dfrac{4x\sqrt{x}}{y^3\sqrt{3y}} \cdot \dfrac{\sqrt{3y}}{\sqrt{3y}} = \dfrac{4x\sqrt{3xy}}{3y^4}$

45. $(3x+5)-(4x+7) = 3x+5-4x-7 = -x-2$

46. $x(1-x)+x[x-(2+x)] = x-x^2+x(-2) = -x^2-x$

47. $(3x^3-4xy-3)+(5xy+x^3+4y-1)$
$= 4x^3+xy+4y-4$

48. $(4xy^3)(6x^4y^2) = 24x^{1+4}y^{3+2} = 24x^5y^5$

49. $(3x-4)(x-1) = 3x^2-3x-4x+4 = 3x^2-7x+4$

50. $(3x-1)(x+2) = 3x^2+6x-x-2 = 3x^2+5x-2$

51. $(4x+1)(x-2) = 4x^2-8x+x-2 = 4x^2-7x-2$

52. $(3x-7)(2x+1) = 6x^2+3x-14x-7$
$= 6x^2-11x-7$

53. $(2x-3)^2 = (2x)^2-2(2x)(3)+3^2 = 4x^2-12x+9$

54. $(4x+3)(4x-3) = 16x^2-9$
Difference of two squares

55.
$$\begin{array}{r} x^2+x-3 \\ \underline{\times\quad 2x^2+1} \\ x^2+x-3 \\ 2x^4+2x^3-6x^2 \\ \hline 2x^4+2x^3-5x^2+x-3 \end{array}$$

56. $(2x-1)^3 = 8x^3-12x^2+6x-1$ Binomial cubed

57.
$$\begin{array}{r} x^2+xy+y^2 \\ \underline{\times\quad x-y} \\ -x^2y-xy^2-y^3 \\ x^3+x^2y+xy^2 \\ \hline x^3 \qquad -y^3 \end{array}$$
Difference of two cubes

58. $\dfrac{4x^2y-3x^3y^3-6x^4y^2}{2x^2y^2} = \dfrac{2}{y}-\dfrac{3xy}{2}-3x^2$

59.
$$\begin{array}{r|l} & 3x^2+2x-3 \\ \cline{2-2} x^2+1 & 3x^4+2x^3 \qquad -x+4 \\ & \underline{3x^4 \qquad +3x^2} \\ & 2x^3-3x^2-x+4 \\ & \underline{2x^3 \qquad +2x} \\ & -3x^2-3x+4 \\ & \underline{-3x^2 \qquad -3} \\ & -3x+7 \end{array}$$
Quotient is $3x^2+2x-3+\dfrac{7-3x}{x^2+1}$.

60.
$$\begin{array}{r|l} & x^3-x^2+2x+7 \\ \cline{2-2} x-3 & x^4-4x^3+5x^2+\quad x \\ & \underline{x^4-3x^3} \\ & -x^3+5x^2 \\ & \underline{-x^3+3x^2} \\ & 2x^2+\quad x \\ & \underline{2x^2-6x} \\ & 7x \\ & \underline{7x-21} \\ & 21 \end{array}$$
Quotient is $x^3-x^2+2x+7+\dfrac{21}{x-3}$.

61. $x^{4/3}(x^{2/3}-x^{-1/3}) = x^{6/3}-x^{3/3} = x^2-x$

62. $\left(\sqrt{x}+\sqrt{a-x}\right)\left(\sqrt{x}-\sqrt{a-x}\right)=\left(\sqrt{x}\right)^2-\left(\sqrt{a-x}\right)^2$
$=x-(a-x)$
$=x-a+x$
$=2x-a$

63. $2x^4-x^3=x^3(2x-1)$

64. $4(x^2+1)^2-2(x^2+1)^3=2(x^2+1)^2[2-(x^2+1)]$
$=2(x^2+1)^2(2-x^2-1)$
$=2(x^2+1)^2(1-x^2)$
$=2(x^2+1)^2(1+x)(1-x)$

65. $4x^2-4x+1=(2x)^2-2(2x)+1^2=(2x-1)^2$

66. $16-9x^2=(4+3x)(4-3x)$

67. $2x^4-8x^2=2x^2(x^2-4)=2x^2(x+2)(x-2)$

68. $x^2-4x-21=(x-7)(x+3)$

69. $3x^2-x-2=(3x+2)(x-1)$

70. $x^2-5x+6=(x-3)(x-2)$

71. $x^2-10x-24=(x-12)(x+2)$

72. $12x^2-23x-24$
Two expressions whose product is $12x^2(-24)=-288x^2$ and whose sum is $-23x$ are $-32x$ and $9x$. So,
$12x^2-23x-24=12x^2+9x-32x-24$
$=3x(4x+3)-8(4x+3)$
$=(4x+3)(3x-8).$

73. $16x^4-72x^2+81=(4x^2)^2-2(4x^2\cdot 9)+9^2$
$=(4x^2-9)^2$
$=[(2x+3)(2x-3)]^2$
$=(2x+3)^2(2x-3)^2$

74. $x^{-2/3}+x^{-4/3}=x^{-4/3}(?)$
$x^{-2/3}+x^{-4/3}=x^{-4/3}(x^{2/3}+1)$
$?=x^{2/3}+1$

75. a. $\dfrac{2x}{2x+4}=\dfrac{2x}{2(x+2)}=\dfrac{x}{x+2}$

b. $\dfrac{4x^2y^3-6x^3y^4}{2x^2y^2-3xy^3}=\dfrac{2x^2y^3(2-3xy)}{xy^2(2x-3y)}$
$=\dfrac{2xy(2-3xy)}{2x-3y}$

76. $\dfrac{x^2-4x}{x^2+4}\cdot\dfrac{x^4-16}{x^4-16x^2}=\dfrac{x(x-4)}{x^2+4}\cdot\dfrac{(x^2-4)(x^2+4)}{x^2(x^2-16)}$
$=\dfrac{(x-4)(x+2)(x-2)}{x(x-4)(x+4)}$
$=\dfrac{(x+2)(x-2)}{x(x+4)}$
$=\dfrac{x^2-4}{x(x+4)}$

77. $\dfrac{x^2+6x+9}{x^2-7x+12}\cdot\dfrac{x^2-3x-4}{x^2+4x+3}$
$=\dfrac{(x+3)(x+3)}{(x-4)(x-3)}\cdot\dfrac{(x-4)(x+1)}{(x+3)(x+1)}=\dfrac{x+3}{x-3}$

78. $\dfrac{x^4-2x^3}{3x^2-x-2}\div\dfrac{x(x^2-4)}{9x^2-4}$
$=\dfrac{x^3(x-2)}{(3x+2)(x-1)}\cdot\dfrac{(3x+2)(3x-2)}{x(x+2)(x-2)}$
$=\dfrac{x^2(3x-2)}{(x-1)(x+2)}$

79. $1+\dfrac{3}{2x}-\dfrac{1}{6x^2}=\dfrac{1}{1}\cdot\dfrac{6x^2}{6x^2}+\dfrac{3}{2x}\cdot\dfrac{3x}{3x}-\dfrac{1}{6x^2}$
$=\dfrac{6x^2+9x-1}{6x^2}$

80. $\dfrac{1}{x-2}-\dfrac{x-2}{4}=\dfrac{1\cdot 4-(x-2)(x-2)}{4(x-2)}=\dfrac{4x-x^2}{4(x-2)}$

81. $\dfrac{x+2}{x(x-1)}-\dfrac{x^2+4}{(x-1)(x-1)}+\dfrac{1}{1}$

$=\dfrac{(x+2)(x-1)-(x^2+4)x+x(x-1)(x-1)}{x(x-1)(x-1)}$

$=\dfrac{x^2+x-2-x^3-4x+x^3-2x^2+x}{x(x-1)^2}$

$=\dfrac{-(x^2+2x+2)}{x(x-1)^2}$

82. $\dfrac{x-1}{x^2-x-2}-\dfrac{x}{x^2-2x-3}+\dfrac{1}{x-2}=\dfrac{x-1}{(x-2)(x+1)}-\dfrac{x}{(x-3)(x+1)}+\dfrac{1}{x-2}$

$=\dfrac{(x-1)(x-3)}{(x-2)(x+1)(x-3)}-\dfrac{x(x-2)}{(x-2)(x+1)(x-3)}+\dfrac{(x+1)(x-3)}{(x-2)(x+1)(x-3)}$

$=\dfrac{x^2-4x+3-x^2+2x+x^2-2x-3}{(x-2)(x+1)(x-3)}$

$=\dfrac{x^2-4x}{(x-2)(x+1)(x-3)}$

$=\dfrac{x(x-4)}{(x-2)(x+1)(x-3)}$

83. $\dfrac{\frac{x-1}{1}-\frac{x-1}{x}}{\frac{1}{x-1}+1}\cdot\dfrac{x(x-1)}{x(x-1)}=\dfrac{x(x-1)^2-(x-1)^2}{x+x(x-1)}$

$=\dfrac{(x-1)^2(x-1)}{x^2}$

$=\dfrac{(x-1)^3}{x^2}$

84. $\dfrac{x^{-2}-x^{-1}}{x^{-2}+x^{-1}}=\dfrac{\frac{1}{x^2}-\frac{1}{x}}{\frac{1}{x^2}+\frac{1}{x}}\cdot\dfrac{x^2}{x^2}=\dfrac{1-x}{1+x}$

85. $\dfrac{3x-3}{\sqrt{x}-1}\cdot\dfrac{\sqrt{x}+1}{\sqrt{x}+1}=\dfrac{3(x-1)(\sqrt{x}+1)}{x-1}=3(\sqrt{x}+1)$

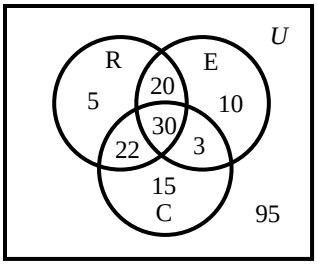
86. $\dfrac{\sqrt{x}-\sqrt{x-4}}{2}\cdot\dfrac{\sqrt{x}+\sqrt{x-4}}{\sqrt{x}+\sqrt{x-4}}=\dfrac{x-(x-4)}{2(\sqrt{x}+\sqrt{x-4})}$

$=\dfrac{x-x+4}{2(\sqrt{x}+\sqrt{x-4})}$

$=\dfrac{4}{2(\sqrt{x}+\sqrt{x-4})}$

$=\dfrac{2}{\sqrt{x}+\sqrt{x-4}}$

87. a. *R*: Recognized
C: Involved
E: Exercised



Numbered statement indicates solution for that question.

1. 30
2. 50 – 30 = 20
3. 52 – 30 = 22
4. 30 + 22 + 20 + $\underline{5}$ = 77
5. 37 – 22 = 15
6. 77 + 15 + $\underline{3}$ = 95

b. $200-(95+5+22+30+20+3+15)=10$ So, 10 exercised only.

c. $63+70-(3+30)=100$ So, 100 exercised or were involved in the community.

88. $-0.75(15)+63.8=52.55\%$

89. $5^2-(5-2)^2=25-3^2=25-9=16$

90. $S=100\left[\dfrac{(1.0075)^n-1}{0.0075}\right]$

a. $S(36)=100\left[\dfrac{(1.0075)^{36}-1}{0.0075}\right]$

$\approx 100\left[\dfrac{0.30865}{0.0075}\right]\approx \$4115.27$

b. $S(240)=100\left[\dfrac{(1.0075)^{240}-1}{0.0075}\right]$

$\approx 100\left[\dfrac{5.00915}{0.0075}\right]\approx \$66,788.69$

91. $C=31.9t+310$

a. $t=2021-2005=16$

b. $C=31.9(16)+310$

$=\$820.40$

c. $4(820.40)=3281.60$

A family of four can expect to pay \$3281.60 for health insurance in 2021.

92. $h = 0.000595s^{1.922}$ or $s = 47.7h^{0.519}$

a. $h = 0.000595(50)^{1.922} \approx 1.1$ inch (about quarter-sized)

b. $s = 47.7(4.5)^{0.519} \approx 104$ mph

93. a.
$$R = 10{,}000\left[\frac{0.0065}{1-(1.0065)^{-n}}\right]$$
$$= 10{,}000\left[\frac{0.0065}{1-\frac{1}{1.0065^n}}\right]\cdot\frac{1.0065^n}{1.0065^n}$$
$$= 10{,}000\left[\frac{0.0065(1.0065)^n}{1.0065^n-1}\right]$$
$$= \frac{65(1.0065)^n}{1.0065^n-1}$$

b. $R = 10{,}000\left[\frac{0.0065}{1-(1.0065)^{-48}}\right] \approx \$243.19$

$R = \frac{65(1.0065)^{48}}{1.0065^{48}-1} \approx \$243.19$

94. $S = kA^{1/3}$

a. $S = k\sqrt[3]{A}$

b. Let S_1 be the number of species on 20,000 acres. Then $S_1 = k\sqrt[3]{20{,}000}$. Let S_2 be the number of species on 45,000 acres. Then
$$S_2 = k\sqrt[3]{45000}$$
$$= \sqrt[3]{2.25\cdot 20{,}000}$$
$$= \sqrt[3]{2.25}\cdot k\sqrt[3]{20{,}000}$$
$$= \sqrt[3]{2.25}\cdot S_1$$
$$S_2 \approx 1.31S_1$$

95. $\text{Profit} = 30x - 0.001x^2 - (300 + 4x)$
$$= -0.001x^2 + 26x - 300$$

96. $\text{Value} = \$1{,}450{,}000 - 0.0025(1{,}450{,}000)x$
$$= \$1{,}450{,}000 - 3625x$$

97. $600 - 13x - 0.5x^2 = 0.5(1200 - 26x - x^2)$ or $0.5(50+x)(24-x)$ or $(25+0.5x)(24-x)$ or $(50+x)(12-0.5x)$

98. a. $C = \frac{1{,}200{,}000}{100-p} - \frac{12{,}000}{1} = \frac{1{,}200{,}000 - 12{,}000(100-p)}{100-p} = \frac{12{,}000p}{100-p}$

b. If $p = 0$, $C = \frac{12{,}000(0)}{100-0} = \frac{0}{100} = \$0$. The cost of removing no pollution is zero.

c. $C = \frac{12{,}000(98)}{100-98} = \$588{,}000$

d. The formula is not defined when $p = 100$. We are dividing by zero. The cost increases as p approaches 100. It is cost prohibitive (or maybe not feasible) to remove all of the pollution.

99. $\frac{1200}{1} + \frac{56x}{1} + \frac{8000}{x} = \frac{1200x}{x} + \frac{56x^2}{x} + \frac{8000}{x} = \frac{56x^2 + 1200x + 8000}{x}$

Chapter 0 Test

1. a. $A = \{6, 8\}$ $B' = \{3, 4, 6\}$
$A \cup B' = \{3, 4, 6, 8\}$

b. {3, 4}, {3, 6}, and {4, 6} are disjoint from B.

c. {6} and {8} are non-empty subsets of A.

2. $(4-2^3)^2 - 3^4\cdot 0^{15} + 12 \div 3 + 1 = (-4)^2 - 0 + 4 + 1$
$= 16 + 4 + 1 = 21$

3. a. $x^4 \cdot x^4 = x^8$

b. $x^0 = 1$, if $x \neq 0$

c. $\sqrt{x} = x^{1/2}$

d. $(x^{-5})^2 = x^{-10}$ or $\frac{1}{x^{10}}$

e. $a^{27} \div a^{-3} = a^{27-(-3)} = a^{30}$

f. $x^{1/2}\cdot x^{1/3} = x^{5/6}$

g. $\frac{1}{\sqrt[3]{x^2}} = \frac{1}{x^{2/3}}$

h. $\frac{1}{x^3} = x^{-3}$

Chapter 0: Algebraic Concepts

4. **a.** $x^{1/5} = \sqrt[5]{x}$

b. $x^{-3/4} = \sqrt[4]{x^{-3}}$ or $\left(\sqrt[4]{x}\right)^{-3}$ or $\dfrac{1}{\sqrt[4]{x^3}}$

5. **a.** $x^{-5} = \dfrac{1}{x^5}$

b. $\left(\dfrac{x^{-8}y^2}{x^{-1}}\right)^{-3} = \dfrac{x^{24}y^{-6}}{x^3} = \dfrac{x^{21}}{y^6}$

6. **a.** $\dfrac{x}{\sqrt{5x}} \cdot \dfrac{\sqrt{5x}}{\sqrt{5x}} = \dfrac{x\sqrt{5x}}{5x} = \dfrac{\sqrt{5x}}{5}$

b. $\sqrt{24a^2b} \cdot \sqrt{a^3b^4} = 2a\sqrt{6b} \cdot ab^2\sqrt{a}$

$= 2a^2b^2\sqrt{6ab}$

c. $\dfrac{1-\sqrt{x}}{1+\sqrt{x}} \cdot \dfrac{1-\sqrt{x}}{1-\sqrt{x}} = \dfrac{1-2\sqrt{x}+x}{1-x}$

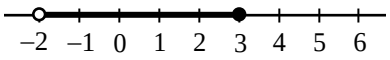
7. $2x^3 - 7x^5 - 5x - 8$

a. Degree is 5.

b. Constant is –8.

c. Coefficient of x is –5.

8. In interval notation, $(-2,\infty) \cap (-\infty,3] = (-2,3]$



9. **a.** $8x^3 - 2x^2 = 2x^2(4x-1)$

b. $x^2 - 10x + 24 = (x-4)(x-6)$

c. $6x^2 - 13x + 6 = (2x-3)(3x-2)$

d. $2x^3 - 32x^5 = 2x^3(1-16x^2)$

$= 2x^3(1-4x)(1+4x)$

10. A quadratic polynomial has degree two.
(c) is the quadratic.
$4 - x - x^2 = 4 - (-3) - (-3)^2 = 4 + 3 - 9 = -2$,
when $x = -3$

11.
$$\begin{array}{r} 2x+1 \\ x^2-1\overline{)\,2x^3+x^2\qquad -7} \\ \underline{2x^3\qquad -2x} \\ x^2+2x-7 \\ \underline{x^2\qquad -1} \\ 2x-6 \end{array}$$

Quotient: $2x + 1 + \dfrac{2x-6}{x^2-1}$

12. **a.** $4y - 5(9-3y) = 4y - 45 + 15y = 19y - 45$

b. $-3t^2(2t^4 - 3t^7) = -6t^6 + 9t^9$

c.
$$\begin{array}{r} x^2-5x+2 \\ \underline{\qquad 4x-1} \\ -x^2+\ 5x-2 \\ \underline{4x^3-20x^2+\ 8x\qquad} \\ 4x^3-21x^2+13x-2 \end{array}$$

d. $(6x-1)(2-3x) = 12x - 18x^2 - 2 + 3x$

$= -18x^2 + 15x - 2$

e. $(2m-7)^2 = 4m^2 - 28m + 49$

f. $\dfrac{x^6}{x^2-9} \cdot \dfrac{x-3}{3x^2} = \dfrac{x^4}{(x+3)(x-3)} \cdot \dfrac{(x-3)}{3}$

$= \dfrac{x^4}{3(x+3)}$

g. $\dfrac{x^4}{9} \div \dfrac{9x^3}{x^6} = \dfrac{x^4}{9} \cdot \dfrac{x^6}{9x^3} = \dfrac{x^7}{81}$

h. $\dfrac{4}{x-8} - \dfrac{x-2}{x-8} = \dfrac{4-x+2}{x-8} = \dfrac{6-x}{x-8}$

i. $\dfrac{x-1}{x^2-2x-3} - \dfrac{3}{x^2-3x}$

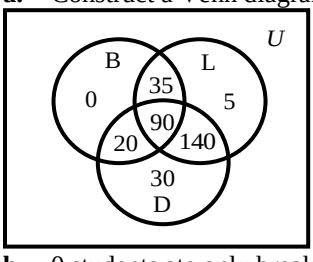
$= \dfrac{x-1}{(x-3)(x+1)} - \dfrac{3}{x(x-3)}$

$= \dfrac{x(x-1)-3(x+1)}{x(x-3)(x+1)}$

$= \dfrac{x^2-x-3x-3}{x(x-3)(x+1)} = \dfrac{x^2-4x-3}{x(x-3)(x+1)}$

13. $\dfrac{\frac{1}{x}-\frac{1}{y}}{\frac{1}{x}+y} \cdot \dfrac{xy}{xy} = \dfrac{y-x}{y+xy^2}$ or $\dfrac{y-x}{y(1+xy)}$

14. **a.** Construct a Venn diagram:



b. 0 students ate only breakfast.

c. 320 – 145 = 175. 175 students skipped breakfast.

15. $S = 1000\left(1 + \dfrac{0.08}{4}\right)^{4x} = 1000\left(1 + \dfrac{0.08}{4}\right)^{4(20)}$

$= 1000(1+0.02)^{80} = 1000(1.02)^{80} \approx 4875.44$

In 20 years, the future value will be about $4875.44.

Chapter 0 Extended Applications & Group Projects

Campaign Management

1. $250,000(0.36) = 90,000$

$50,000(0.3) = 15,000$

$90,000 - 15,000 = 75,000$

So 75,000 voters read the newspaper but do not watch the local cable network news.

2. 75,000 newspaper
35,000 cable news
15,000 both
125,000

125,000 read the newspaper or watch cable news or both.

3.

	Number of Voters Reached	Total Cost	Cost per Voter Reached
Pamphlet	125,000	$112,500	$0.90
Cable News	50,000	$40,000	$0.80
Newspaper	90,000	$27,000	$0.30

4. Since 125,000 voters are reached just through newspaper and cable network news advertising, and since reaching voters through each of these means is less expensive than advertising via pamphlet, one plan might be to pay $40,000 + $27,000 = $67,000 to reach voters through the cable network news and advertising alone (and thus not use a pamphlet).